$$
\begin{aligned}
\mathbf{e}^x &= \operatorname{ch}(x) + \operatorname{sh}(x) \\
1 &= \operatorname{ch}^2(x) - \operatorname{sh}^2(x) \\
\operatorname{ch}(x+y) &= \operatorname{ch}(x)\operatorname{ch}(y) + \operatorname{sh}(x)\operatorname{sh}(y) \\
\operatorname{sh}(x+y) &= \operatorname{sh}(x)\operatorname{ch}(y) + \operatorname{ch}(x)\operatorname{sh}(y) \\
\operatorname{ch}(2x) &= 2\operatorname{ch}^2(x) - 1 = 1 + 2\operatorname{sh}^2(x) \\
\operatorname{sh}(2x) &= 2\operatorname{sh}(x)\operatorname{ch}(x)
\end{aligned}
$$

$$
\begin{aligned}
\operatorname{ch}(x)\operatorname{ch}(y) &= \frac{1}{2}[\operatorname{ch}(x+y) + \operatorname{ch}(x-y)] \\
\operatorname{sh}(x)\operatorname{sh}(y) &= \frac{1}{2}[\operatorname{ch}(x+y) - \operatorname{ch}(x-y)] \\
\operatorname{sh}(x)\operatorname{ch}(y) &= \frac{1}{2}[\operatorname{sh}(x+y) + \operatorname{sh}(x-y)]
\end{aligned}
$$

$$
\begin{aligned}
\operatorname{ch}(x) + \operatorname{ch}(y) &= 2\operatorname{ch}(\frac{x+y}{2})\operatorname{ch}(\frac{x-y}{2}) \\
\operatorname{ch}(x) - \operatorname{ch}(y) &= 2\operatorname{sh}(\frac{x+y}{2})\operatorname{sh}(\frac{x-y}{2}) \\
\operatorname{sh}(x) + \operatorname{sh}(y) &= 2\operatorname{sh}(\frac{x+y}{2})\operatorname{ch}(\frac{x-y}{2}) \\
\operatorname{sh}(x) - \operatorname{sh}(y) &= 2\operatorname{sh}(\frac{x-y}{2})\operatorname{ch}(\frac{x+y}{2}) \\
\operatorname{th}(x+y) &= \frac{\operatorname{th}(x) + \operatorname{th}(y)}{1 + \operatorname{th}(x)\operatorname{th}(y)}
\end{aligned}
$$