Maximal wild monodromy in unequal characteristic

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Abstract

Let R be a complete discrete valuation ring of mixed characteristic $(0, p)$ with fraction field K. We study stable models of p-cyclic covers of $\mathbb{P}^1_K$. First, we determine the monodromy extension, the monodromy group, its filtration and the Swan conductor for special covers of arbitrarily high genus with potential good reduction. In the case $p = 2$ we consider hyperelliptic curves of genus 2.

1 Introduction

Let (R, v) be a complete discrete valuation ring of mixed characteristic $(0, p)$ with fraction field K containing a primitve p-th root of unity ζ_p and algebraically closed residue field k. The stable reduction theorem states that given a smooth, projective, geometrically connected curve C/K of genus $g(C) \geq 2$, there exists a unique minimal Galois extension M/K called *the monodromy extension of* C/K such that $C_M := C \times M$ has stable reduction over M. The group $G = \mathrm{Gal}(M/K)$ is the *monodromy group of* C/K. In a previous paper, Lehr and Matignon [LM06] gave an algorithm to determine the stable reduction of p-cyclic covers of $\mathbb{P}^1_K$ under the extra assumption of *equidistant geometry* of the branch locus and obtain information about the monodromy extension M/K of C/K. This makes effective a theorem of Raynaud [Ray90] in the case of p-cyclic covers of $\mathbb{P}^1_K$. The present article studies examples of such p-cyclic covers but is independent of their work and develops specific methods to treat our special covers.

Let $\mathcal{C}$ be the stable model of C_M/M and $\mathrm{Aut}_k(\mathcal{C}_k)^\#$ the subgroup of $\mathrm{Aut}_k(\mathcal{C}_k)$ of elements acting trivially on the reduction in $\mathcal{C}_k$ of the ramification locus of $C_M \to \mathbb{P}^1_M$ (see [Liu02] 10.1.3 for the definition of the reduction

map of C_M). One derives from the stable reduction theorem the following injection :

$$\mathrm{Gal}(M/K) \hookrightarrow \mathrm{Aut}_k(\mathcal{C}_k)^{\#}. \tag{1}$$

When the p-Sylow subgroups of these groups are isomorphic, one says that the *wild monodromy is maximal*. We are interested in realization of covers such that the p-adic valuation of $|\mathrm{Aut}_k(\mathcal{C}_k)^{\#}|$ is large and having maximal wild monodromy, we will study ramification filtrations and Swan conductors of their monodromy extensions.

In section 3, we consider examples of covers of arbitrarily high genus having potential good reduction. Let $n \in \mathbb{N}^{\times}$, $q = p^n$, $\lambda = \zeta_p - 1$ and $K = \mathbb{Q}_p^{\mathrm{ur}}(\lambda^{1/(1+q)})$. We study covers C_c/K of $\mathbb{P}^1_K$ defined by $Y^p = 1 + cX^q + X^{1+q}$ with $c \in R$, $v(\lambda^{p/(1+q)}) > v(c)$ and $v(c^p - c) \geq v(p)$.

Theorem 1.1. *The stable reduction $\mathcal{C}_k/k$ is canonically a p-cyclic cover of $\mathbb{P}^1_k$. It is smooth, ramified at one point ∞ and étale outside ∞. The ramification locus of $C_M \to \mathbb{P}^1_M$ reduces in ∞ and the group $\mathrm{Aut}_k(\mathcal{C}_k)^{\#}$ has a unique p-Sylow subgroup $\mathrm{Aut}_k(\mathcal{C}_k)^{\#}_1$. Moreover, the curve C_c/K has maximal wild monodromy M/K. The extension M/K is the decomposition field of an explicitly given polynomial and $\mathrm{Gal}(M/K) \simeq \mathrm{Aut}_k(\mathcal{C}_k)^{\#}_1$ is an extra-special p-group of order pq^2.*

Let X/k be a p-cyclic cover of $\mathbb{P}^1_k$ of genus $g(X)$, ramified at one point ∞ and étale outside ∞. According to [LM05], the p-Sylow subgroup $G_{\infty,1}(X)$ of the subgroup of $\mathrm{Aut}_k(X)$ of automorphisms leaving ∞ fixed satisfies $|G_{\infty,1}(X)| \leq \frac{4p}{(p-1)^2} g(X)^2$. The stable reduction $\mathcal{C}_k/k$ of Theorem 1.1 is such that $G_{\infty,1}(\mathcal{C}_k) = \mathrm{Aut}_k(\mathcal{C}_k)^{\#}_1$ and $|G_{\infty,1}(\mathcal{C}_k)| = \frac{4p}{(p-1)^2} g(\mathcal{C}_k)^2$. So we obtain the largest possible maximal wild monodromy for curves over some finite extension of $\mathbb{Q}_p^{\mathrm{ur}}$ with genus in $\frac{p-1}{2}p^{\mathbb{N}}$ in the good reduction case.

The group $G_{\infty,1}(\mathcal{C}_k) = \mathrm{Aut}_k(\mathcal{C}_k)^{\#}_1$ is endowed with the ramification filtration $(G_{\infty,i}(\mathcal{C}_k))_{i\geq 0}$ which is easily seen to be :

$$G_{\infty,0}(\mathcal{C}_k) = G_{\infty,1}(\mathcal{C}_k) \supsetneq \mathrm{Z}(G_{\infty,0}(\mathcal{C}_k)) = G_{\infty,2}(\mathcal{C}_k) = \cdots = G_{\infty,1+q}(\mathcal{C}_k) \supsetneq \{1\}.$$

Moreover, $G := \mathrm{Gal}(M/K)$ being the Galois group of a finite extension of $\mathbb{Q}_p^{\mathrm{ur}}$, it is endowed with the ramification filtration $(G_i)_{i\geq 0}$ of an arithmetic nature. Since $G \simeq G_{\infty,1}(\mathcal{C}_k)$ it is natural to ask for the behaviour of $(G_i)_{i\geq 0}$ under (1), that is to compare $(G_i)_{i\geq 0}$ and $(G_{\infty,i}(\mathcal{C}_k))_{i\geq 0}$. One shows that they actually coincide and we compute the conductor exponent $f(\mathrm{Jac}(C_c)/K)$ of $\mathrm{Jac}(C_c)/K$ and its Swan conductor $\mathrm{sw}(\mathrm{Jac}(C_c)/K)$:

Theorem 1.2. *Under the hypotheses of Theorem 1.1, the lower ramification filtration of G is :*

$$G = G_0 = G_1 \supsetneq \mathrm{Z}(G) = G_2 = \cdots = G_{1+q} \supsetneq \{1\}.$$

Then, $f(\mathrm{Jac}(C_c)/K) = (2q+1)(p-1)$ and, in the case where $c \in \mathbb{Q}_p^{\mathrm{ur}}$, $\mathrm{sw}(\mathrm{Jac}(C_c)/\mathbb{Q}_p^{\mathrm{ur}}) = 1$.

The value $\mathrm{sw}(\mathrm{Jac}(C_c)/\mathbb{Q}_p^{\mathrm{ur}}) = 1$ is the smallest one among abelian varieties over $\mathbb{Q}_p^{\mathrm{ur}}$ with non tame monodromy extension. That is, in some sense, a counter part of [BK94] and [LRS93] where an upper bound for the conductor exponent is given and it is shown that this bound is actually achieved.

In section 4, one restricts to the case $p = 2$ and genus 2. In this situation there are three possible types of geometry for the stable reduction. In each case, one gives a family of curves with this degeneration type such that the wild monodromy is maximal. This has applications to the inverse Galois problem. For example, we have the following :

Proposition 1.1. *Let $K = \mathbb{Q}_2^{\mathrm{ur}}(2^{1/15})$ and C_0/K the smooth, projective, geometrically integral curve given by $Y^2 = 1 + 2^{3/5}X^2 + X^3 + 2^{2/5}X^4 + X^5$. The irreducible components of its stable reduction $\mathcal{C}_k/k$ are elliptic curves. The monodromy extension M/K of C_0/K is the decomposition field of an explicitly given polynomial. The curve C_0/K has maximal wild monodromy and $G := \mathrm{Gal}(M/K) \simeq Q_8 \times Q_8$. Moreover, we have*

$$G_i \simeq \begin{cases} Q_8 \times Q_8, & -1 \leq i \leq 1, \\ \mathrm{Z}(Q_8) \times Q_8, & 2 \leq i \leq 3, \\ \{1\} \times Q_8 & 4 \leq i \leq 31, \\ \{1\} \times \mathrm{Z}(Q_8), & 32 \leq i \leq 543, \\ \{1\} \times \{1\}, & 544 \leq i. \end{cases}$$

and $\mathrm{sw}(\mathrm{Jac}(C_0)/K) = 45$.

Some of the results that we give here were already available in a previous preprint of C. Lehr and M. Matignon (see [LM]) , results about the arithmetic of the monodromy extensions, ramification and conductors are new.

2 Background.

Notations. Let (R, v) be a complete discrete valuation ring (DVR) of mixed characteristic $(0, p)$ with fraction field K and algebraically closed residue field k. We denote by π_K a uniformizer of R and assume that K contains a primitive p-th root of unity ζ_p. Let $\lambda := \zeta_p - 1$. If L/K is an algebraic extension,

we will denote by π_L(resp. v_L, resp. L°) a uniformizer for L (resp. the prolongation of v to L such that $v_L(\pi_L) = 1$, resp. the ring of integers of L). If there is no possible confusion we note v for the prolongation of v to an algebraic closure K^{alg} of K.

1. *Stable reduction of curves.* The first result is due to Deligne and Mumford (see for example [Liu02] for a presentation following Artin and Winters).

Theorem 2.1 (Stable reduction theorem)**.** *Let C/K be a smooth, projective, geometrically connected curve over K of genus $g(C) \geq 2$. There exists a unique finite Galois extension M/K minimal for the inclusion relation such that C_M/M has stable reduction. The stable model $\mathcal{C}$ of C_M/M over M° is unique up to isomorphism. One has a canonical injective morphism :*

$$\text{Gal}(M/K) \overset{i}{\hookrightarrow} \text{Aut}_k(\mathcal{C}_k). \tag{2}$$

Remarks :

1. Let's explain the action of $\text{Gal}(K^{\text{alg}}/K)$ on $\mathcal{C}_k/k$. The group $\text{Gal}(K^{\text{alg}}/K)$ acts on $C_M := C \times M$ on the right. By unicity of the stable model, this action extends to $\mathcal{C}$:

$$\begin{array}{ccc} \mathcal{C} & \xrightarrow{\sigma} & \mathcal{C} \\ \downarrow & & \downarrow \\ M^\circ & \xrightarrow{\sigma} & M^\circ \end{array}$$

   Since $k = k^{\text{alg}}$ one gets $\sigma \times k = \text{Id}_k$, whence the announced action. The last assertion of the theorem characterizes the elements of $\text{Gal}(K^{\text{alg}}/M)$ as the elements of $\text{Gal}(K^{\text{alg}}/K)$ that trivially act on $\mathcal{C}_k/k$.

2. If $p > 2g(C)+1$, then C/K has stable reduction over a tamely ramified extension of K. We will study examples of covers with $p \leq 2g(C) + 1$.

3. Our results will cover the elliptic case. Let E/K be an elliptic curve with additive reduction. If its modular invariant is integral, then there exists a smallest extension M of K over which E/K has good reduction. Else E/K obtains split multiplicative reduction over a unique quadratic extension of K (see [Kra90]).

Definition 2.1. *The extension M/K is the monodromy extension of C/K. We call* $\mathrm{Gal}(M/K)$ *the monodromy group of C/K. It has a unique p-Sylow subgroup* $\mathrm{Gal}(M/K)_1$ *called the wild monodromy group. The extension $M/M^{\mathrm{Gal}(M/K)_1}$ is the wild monodromy extension.*

From now on we consider smooth, projective, geometrically integral curves C/K of genus $g(C) \geq 2$ birationally given by $Y^p = f(X) := \prod_{i=0}^{t}(X - x_i)^{n_i}$ with $(p, \sum_{i=0}^{t} n_i) = 1$, $(p, n_i) = 1$ and $\forall\ 0 \leq i \leq t, x_i \in R^\times$. Moreover, we assume that $\forall i \neq j,\ v(x_i - x_j) = 0$, that is to say, the branch locus $B = \{x_0, \ldots, x_t, \infty\}$ of the cover has *equidistant geometry*. We denote by *Ram* the ramification locus of the cover.

Remark : We only ask p-cyclic covers to satisfy Raynaud's theorem 1' [Ray90] condition, that is the branch locus is K-rational with equidistant geometry. This has consequences on the image of (2).

Proposition 2.1. *Let $\mathcal{T} = \mathrm{Proj}(M^\circ[X_0, X_1])$ with $X = X_0/X_1$. The normalization $\mathcal{Y}$ of $\mathcal{T}$ in $K(C_M)$ admits a blowing-up $\tilde{\mathcal{Y}}$ which is a semi-stable model of C_M/M. The dual graph of $\tilde{\mathcal{Y}}_k/k$ is a tree and the points in Ram specialize in a unique irreducible component $D_0 \simeq \mathbb{P}^1_k$ of $\tilde{\mathcal{Y}}_k/k$. There exists a contraction morphism $h : \tilde{\mathcal{Y}} \to \mathcal{C}$, where $\mathcal{C}$ is the stable model of C_M/M and*

$$\mathrm{Gal}(M/K) \hookrightarrow \mathrm{Aut}_k(\mathcal{C}_k)^{\#}, \tag{3}$$

where $\mathrm{Aut}_k(\mathcal{C}_k)^{\#}$ *is the subgroup of* $\mathrm{Aut}_k(\mathcal{C}_k)$ *of elements inducing the identity on $h(D_0)$.*

Proof. Let $f(X) = (X - x_0)^{n_0} S(X)$ and $an_0 + bp = 1$. Then above $\mathcal{T}\backslash B = \mathrm{Spec}\, A$ (resp. $\mathcal{T}\backslash\{B\backslash x_0\} = \mathrm{Spec}\, A_0$), the equation of $\mathcal{Y}$ is :

$$A[Y]/(Y^p - f(X)) \ \ (\text{resp. } A_0[Y]/(Y^p - (X - x_0)S(X)^a)),$$

(using [Liu02] 4.1.18). Since $v(S(x_0)) = 0$, the ramification locus *Ram* specialize in a unique component D_0 of $\tilde{\mathcal{Y}}_k$. Using [Ray90] theorem 2, one sees that $\tilde{\mathcal{Y}}_k$ is a tree. It implies that there exists a contraction morphism $h : \tilde{\mathcal{Y}} \to \mathcal{X}$ of the components of $\tilde{\mathcal{Y}}_k$ isomorphic to $\mathbb{P}^1_k$ meeting $\tilde{\mathcal{Y}}_k$ in at most 2 points ([Liu02] 7.5.4 and 8.3.36). The scheme $\mathcal{X}$ is seen to be stable ([Liu02] 10.3.31), so $\mathcal{X} \simeq \mathcal{C}$.

The component D_0 is smooth of genus 0 (being birational to a curve with function field a purely inseparable extension of $K(\mathbb{P}^1_k)$) so $D_0 \simeq \mathbb{P}^1_k$. Then, B having K-rational equidistant geometry with $|B| \geq 3$, any element of $\mathrm{Gal}(M/K)$ induces the identity on D_0, giving (3). □

Remark : The component D_0 is the so called *original component*.

Definition 2.2. *If* (3) *is surjective, we say that* C *has* maximal monodromy. *If* $v_p(|\operatorname{Gal}(M/K)|) = v_p(|\operatorname{Aut}_k(\mathcal{C}_k)^{\#}|)$, *we say that* C *has* maximal wild monodromy.

Definition 2.3. *The valuation on* $K(X)$ *corresponding to the discrete valuation ring* $R[X]_{(\pi_K)}$ *is called the* Gauss valuation v_X *with respect to* X. *We then have*

$$v_X\left(\sum_{i=0}^{m} a_i X^i\right) = \min\{v(a_i),\ 0 \leq i \leq m\}.$$

Note that a change of variables $T = \frac{X-y}{\rho}$ *for* $y, \rho \in R$ *induces a Gauss valuation* v_T. *These valuations are exactly those that come from the local rings at generic points of components in the semi-stables models of* $\mathbb{P}^1_K$.

2. *Galois extensions of complete DVRs.* Let L/K be a finite Galois extension with group G. Then G is endowed with a *lower ramification filtration* $(G_i)_{i\geq -1}$ where G_i is the *i-th lower ramification group* defined by $G_i := \{\sigma \in G \mid v_L(\sigma(\pi_L) - \pi_L) \geq i+1\}$. The integers i such that $G_i \neq G_{i+1}$ are called *lower breaks*. For $\sigma \in G - \{1\}$, let $i_G(\sigma) := v_L(\sigma(\pi_L) - \pi_L)$. The group G is also endowed with a *higher ramification filtration* $(G^i)_{i\geq -1}$ which can be computed from the G_i's by means of the *Herbrand's function* $\varphi_{L/K}$. The real numbers t such that $\forall \epsilon > 0,\ G^{t+\epsilon} \neq G^t$ are called *higher breaks*. We will use the following lemma (see for example [Hyo87]).

Lemma 2.1. *Let* L/K *defined by* $X^p = 1 + w\pi_K^s$ *with* $0 < s < \frac{p}{p-1} v_K(p)$, $(s,p) = 1$ *and* $w \in R^\times$. *The different ideal* $\mathcal{D}_{L/K}$ *satisfies :*

$$v_K(\mathcal{D}_{L/K}) = v_K(p) + \frac{p-1}{p}(1-s).$$

3. *Extra-special p-groups.* The Galois groups and automorphism groups that we will have to consider are p-groups with peculiar group theoretic properties (see for example [Suz82] for an account on extra-special p-groups). We will denote by $\mathrm{Z}(G)$ (resp. $\mathrm{D}(G)$, $\Phi(G)$) the center (resp. the derived subgroup, the Frattini subgroup) of G. If G is a p-group, one has $\Phi(G) = \mathrm{D}(G)G^p$.

Definition 2.4. *An extra-special p-group is a non abelian p-group* G *such that* $\mathrm{D}(G) = \mathrm{Z}(G) = \Phi(G)$ *has order* p.

Proposition 2.2. *Let* G *be an extra-special p-group.*

1. *Then* $|G| = p^{2n+1}$ *for some* $n \in \mathbb{N}^\times$.

2. One has the exact sequence

$$0 \to \mathrm{Z}(G) \to G \to (\mathbb{Z}/p\mathbb{Z})^{2n} \to 0.$$

Remark : With the previous notations, we will encounter curves such that the p-Sylow subgroup of $\mathrm{Aut}_k(\mathcal{C}_k)^{\#}$ is an extra-special p-group. In this case, the above short exact sequence has a geometric description that we will make explicit later on.

4. *Torsion points on abelian varieties.* Let A/K be an abelian variety over K with potential good reduction. Let $\ell \neq p$ be a prime number, we denote by $A[\ell]$ the ℓ-torsion subgroup of $A(K^{\mathrm{alg}})$ and by $T_\ell(A) = \varprojlim A[\ell^n]$ (resp. $V_\ell(A) = T_\ell(A) \otimes \mathbb{Q}_\ell$) the Tate module (resp. ℓ-adic Tate module) of A.

The following result may be found in [Gur03] (paragraph 3). We recall it for the convenience of the reader.

Lemma 2.2. *Let $k = k^{\mathrm{alg}}$ be a field with* $\mathrm{char}\ k = p \geq 0$ *and C/k be a projective, smooth, integral curve. Let $\ell \neq p$ be a prime number and H be a finite subgroup of* $\mathrm{Aut}_k(C)$ *such that $(|H|, \ell) = 1$. Then*

$$2g(C/H) = \dim_{\mathbb{F}_\ell} \mathrm{Jac}(C)[\ell]^H$$

.

If $\ell \geq 3$, then $L = K(A[\ell])$ is the minimal extension over which A/K has good reduction. It is a Galois extension with group G (see [ST68]). We denote by r_G (resp. 1_G) the character of the regular (resp. unit) representation of G. We denote by I the inertia group of K^{alg}/K. For further explanations about conductor exponents see [Ser67], [Ogg67] and [ST68]. We assume that L/K is totally ramified.

Definition 2.5. *1. Let*

$$a_G(\sigma) := -i_G(\sigma), \quad \sigma \neq 1,$$
$$a_G(1) := \sum_{\sigma \neq 1} i_G(\sigma),$$

and $\mathrm{sw}_G := a_G - r_G + 1_G$. *Then, a_G is the character of a $\mathbb{Q}_\ell[G]$-module and there exists a projective $\mathbb{Z}_\ell[G]$-module* Sw_G *such that* $\mathrm{Sw}_G \otimes_{\mathbb{Z}_\ell} \mathbb{Q}_\ell$ *has character* sw_G.

2. *We still denote by $T_\ell(A)$ (resp. $A[\ell]$) the $\mathbb{Z}_\ell[G]$-module (resp. $\mathbb{F}_\ell[G]$-module) afforded by $G \to \mathrm{Aut}(T_\ell(A))$ (resp. $G \to \mathrm{Aut}(A[\ell])$). Let*

$$\mathrm{sw}(A/K) := \dim_{\mathbb{F}_\ell} \mathrm{Hom}_G(\mathrm{Sw}_G, A[\ell]),$$
$$\epsilon(A/K) := \mathrm{codim}_{\mathbb{Q}_\ell} V_\ell(A)^I.$$

The integer $f(A/K) := \epsilon(A/K) + \mathrm{sw}(A/K)$ is the so called conductor exponent of A/K and $\mathrm{sw}(A/K)$ is the Swan conductor of A/K.

Proposition 2.3. *Let $\ell \neq p$, $\ell \geq 3$ be a prime number.*

1. *The integers $\mathrm{sw}(A/K)$ and $\epsilon(A/K)$ are independent of ℓ.*
2. *One has*
$$\mathrm{sw}(A/K) = \sum_{i \geq 1} \frac{|G_i|}{|G_0|} \dim_{\mathbb{F}_\ell} A[\ell]/A[\ell]^{G_i}.$$
Moreover, for ℓ large enough, $\epsilon(A/K) = \dim_{\mathbb{F}_\ell} A[\ell]/A[\ell]^{G_0}$.

Remark : It follows from the definition that $\mathrm{sw}(A/K) = 0$ if and only if $G_1 = \{1\}$. The Swan conductor is a measure of the wild ramification.

3 Covers with potential good reduction.

We start by fixing notations that will be used throughout this section.

Notations. We denote by $\mathfrak{m}$ the maximal ideal of $(K^{\mathrm{alg}})^\circ$. Let $n \in \mathbb{N}^\times$, $q = p^n$ and $a_n = (-1)^q(-p)^{p+p^2+\cdots+q}$. We denote by $\mathbb{Q}_p^{\mathrm{ur}}$ the maximal unramified extension of $\mathbb{Q}_p$. Let $K := \mathbb{Q}_p^{\mathrm{ur}}(\lambda^{1/(1+q)})$. For $c \in R$ let

$$f_{q,c}(X) = 1 + cX^q + X^{1+q}.$$

One defines the *modified monodromy polynomial* by

$$L_c(X) = X^{q^2} - a_n(c+X)f_{q,c}(X)^{q-1}.$$

Let C_c/K and A_q/k be the smooth projective integral curves birationally given respectively by $Y^p = f_{q,c}(X)$ and $w^p - w = t^{1+q}$.

Theorem 3.1. *The curve C_c/K has potential good reduction isomorphic to A_q/k.*

1. *If $v(c) \geq v(\lambda^{p/(1+q)})$, then the monodromy extension of C_c/K is trivial.*

2. If $v(c) < v(\lambda^{p/(1+q)})$, let y be a root of $L_c(X)$ in K^{alg}. Then C_c has good reduction over $K(y, f_{q,c}(y)^{1/p})$. If $L_c(X)$ is irreducible over K, then C_c/K has maximal wild monodromy. The monodromy extension of C_c/K is $M = K(y, f_{q,c}(y)^{1/p})$ and $G = \mathrm{Gal}(M/K)$ is an extra-special p-group of order pq^2. If $c \in R$ with $v(c^p - c) \geq v(p)$, then $L_c(X)$ is irreducible over K, the lower ramification filtration of G is

$$G = G_0 = G_1 \supsetneq G_2 = \cdots = G_{1+q} = \mathrm{Z}(G) \supsetneq \{1\}.$$

Moreover, one has $f(\mathrm{Jac}(C_c)/K) = (2q+1)(p-1)$. If $c \in \mathbb{Q}_p^{\mathrm{ur}}$ then $\mathrm{sw}(\mathrm{Jac}(C_c)/\mathbb{Q}_p^{\mathrm{ur}}) = 1$.

Proof. 1. Assume that $v(c) \geq v(\lambda^{p/(1+q)})$. Set $\lambda^{p/(1+q)}T = X$ and $\lambda W + 1 = Y$. Then, the equation defining C_c/K becomes

$$(\lambda W + 1)^p = \sum_{i=0}^{p} \binom{p}{i} \lambda^i W^i = 1 + c\lambda^{pq/(1+q)} T^q + \lambda^p T^{1+q}.$$

After simplification by λ^p and reduction modulo π_K this equation gives :

$$w^p - w = at^q + t^{1+q},\ a \in k. \tag{4}$$

By Hurwitz formula the genus of the curve defined by (4) is seen to be that of C_c/K . Applying [Liu02] 10.3.44, there is a component in the stable reduction birationally given by (4). The stable reduction being a tree, the curve C_c/K has good reduction over K.

2. The proof of the first part is divided into six steps. Let y be a root of $L_c(X)$.

Step I : *One has $v(y) = v(a_n c)/q^2$.*
Since y is a root of $L_c(X)$, one has

$$y^{q^2} = a_n(c+y) f_{q,c}(y)^{q-1},$$

so $v(y) > 0$. Assume that $v(c+y) \geq v(y)$. Then, $q^2 v(y) \geq v(a_n) + v(y)$ and $v(c) \geq v(y) \geq \frac{v(a_n)}{q^2-1} = \frac{p}{q+1} v(\lambda)$, which is a contradiction. So $v(c+y) < v(y)$ thus $v(c+y) = v(c)$.

Step II : *Define S and T by $\lambda^{p/(1+q)}T = (X - y) = S$. Then,*

$$f_{q,c}(S+y) \equiv f_{q,c}(y) + y^q S + (c+y)S^q + S^{1+q} \mod \lambda^p \mathfrak{m}[T].$$

Using the following formula for $A \in K^{\mathrm{alg}}$ with $v(A) > 0$ and $B \in (K^{\mathrm{alg}})^\circ[T]$

$$(A+B)^q \equiv (A^{q/p} + B^{q/p})^p \mod p^2\mathfrak{m}[T],$$

one computes $\mod \lambda^p\mathfrak{m}[T]$

$$\begin{aligned} f_{q,c}(y+S) &= 1 + c(y+S)^q + (y+S)^{1+q} \\ &\equiv 1 + c(y^{q/p} + S^{q/p})^p + (y+S)(y^{q/p} + S^{q/p})^p \\ &\equiv f_{q,c}(y) + (y^q + \Sigma)S + (c+y)S^q + S^{1+q} + (c+y)\Sigma, \end{aligned}$$

where $\Sigma = \sum_{k=1}^{p-1} \binom{p}{k} y^{kq/p} S^{(p-k)q/p}$. Using **Step I**, one checks that $\Sigma \in \lambda^p\mathfrak{m}[T]$.

Step III : *Let $R_1 := K[y]^\circ$. For all $0 \leq i \leq n$, there exist $B_i \in R_1$ and $A_i(S) \in R_1[S]$ such that* $\mod \lambda^p\mathfrak{m}[T]$ *one has :*

$$f_{q,c}(S+y) \equiv f_{q,c}(y)(1 + SA_i(S))^p + y^q S + B_i S^{q/p^i} + S^{1+q}. \tag{5}$$

One defines the $A_i(S)$'s and the B_i's by induction. For all $0 \leq i \leq n-1$, let

$$\begin{aligned} B_n &:= -y^q \quad \text{and} \quad B_i := f_{q,c}(y)\frac{B_{i+1}^p}{(-pf_{q,c}(y))^p}, \\ A_0(S) &:= 0 \quad \text{and} \quad SA_{i+1}(S) := SA_i(S) - p^{-1}f_{q,c}(y)^{-1}B_{i+1}S^{q/p^{i+1}}. \end{aligned}$$

One checks that for all $0 \leq i \leq n$

$$B_i/f_{q,c}(y) = (-p)(-p)^{-1-p-\cdots-p^{n-i}}(-y^q/f_{q,c}(y))^{p^{n-i}},$$

and

$$v(B_i) \geq (1 + \frac{1}{p} + \cdots + \frac{1}{p^{i-1}})v(p), \ \forall 1 \leq i \leq n. \tag{6}$$

It follows that $\forall\ 1 \leq i \leq n$, $p^{-1}B_i \in R_1$, $\forall\ 0 \leq i \leq n$ $A_i(S) \in R_1[S]$ and $B_0 = c + y$ since $L_c(y) = 0$.

One proves this step by induction on i. According to **Step II**, the equation (5) holds for $i = 0$. Assume that (5) is satisfied for i. Taking into account that

$$\begin{aligned} &f_{q,c}(y)(1 + (-1)^p)\frac{B_{i+1}^p}{p^p f_{q,c}(y)^p}S^{q/p^i}, \\ &\sum_{k=2}^{p-1}\binom{p}{k}(\frac{B_{i+1}}{pf_{q,c}(y)}S^{q/p^{i+1}})^k(1 + SA_{i+1}(S))^{p-k}, \\ &B_{i+1}S^{q/p^{i+1}}\sum_{k=1}^{p-1}\binom{p-1}{k}S^k A_{i+1}(S)^k, \end{aligned}$$

are in $\lambda^p\mathfrak{m}[T]$, one gets (5) for $i+1$.

Step IV : *The curve C_c/K has good reduction over $K(y, f_{q,c}(y)^{1/p})$.*
Applying **Step III** for $i = n$, one gets

$$f_{q,c}(y+S) \equiv f_{q,c}(y)(1+SA_n(S))^p + S^{1+q} \mod \lambda^p\mathfrak{m}[T],$$

then the change of variables in $K(y, f_{q,c}(y)^{1/p})$

$$X = \lambda^{p/(1+q)}T + y = S + y \text{ and } \frac{Y}{f_{q,c}(y)^{1/p}} = \lambda W + 1 + SA_n(S),$$

induces in reduction $w^p - w = t^{1+q}$ with genus $g(C_c)$. So [Liu02] 10.3.44 implies that the above change of variables gives the stable model.

Step V : *For any distinct roots y_i, y_j of $L_c(X)$, $v(y_i - y_j) = v(\lambda^{p/(1+q)})$.*
The changes of variables $T = (X - y_i)/\lambda^{p/(1+q)}$ and $T = (X - y_j)/\lambda^{p/(1+q)}$ induce equivalent Gauss valuations of $K(C_c)$ else applying [Liu02] 10.3.44 would contradict the uniqueness of the stable model. In particular $v(y_i - y_j) \geq v(\lambda^{p/(1+q)})$.

Using **Step I**, one checks that $v(q^2y^{q^2-1}) > v(a_n)$ and $v(f'_{q,c}(y)) > 0$, so :

$$v(L'_c(y)) = v(a_n) = (q^2-1)v(\lambda^{p/(1+q)}).$$

Taking into account that $L'_c(y_i) = \prod_{j\neq i}(y_i - y_j)$ and $\deg L_c(X) = q^2$, one obtains $v(y_i - y_j) = v(\lambda^{p/(1+q)})$.

Step VI : *If $L_c(X)$ is irreducible over K, then $K(y, f_{q,c}(y)^{1/p})$ is the monodromy extension M of C_c/K and $G := \mathrm{Gal}(M/K)$ is an extra-special p-group of order pq^2.*
Let $(y_i)_{i=1,\dots,q^2}$ be the roots of $L_c(X)$, $L := K(y_1, \dots, y_{q^2})$ and M/K be the monodromy extension of C_c/K. Any $\tau \in \mathrm{Gal}(L/K) - \{1\}$ is such that $\tau(y_i) = y_j$ for some $i \neq j$. Thus, the change of variables

$$X = \lambda^{p/(1+q)}T + y_i \text{ and } \frac{Y}{f_{q,c}(y_i)^{1/p}} = \lambda W + 1 + SA_n(S),$$

induces the stable model and τ acts on it by :

$$\tau(T) = \frac{X - y_j}{\lambda^{p/(1+q)}}, \quad \text{hence} \quad T - \tau(T) = \frac{y_j - y_i}{\lambda^{p/(1+q)}}.$$

According to **Step V**, τ acts non-trivially on the stable reduction. It follows that $L \subseteq M$. Indeed if $\mathrm{Gal}(K^{\mathrm{alg}}/M) \not\subseteq \mathrm{Gal}(K^{\mathrm{alg}}/L)$ it would exist

$\sigma \in \mathrm{Gal}(K^{\mathrm{alg}}/M)$ inducing $\bar{\sigma} \neq \mathrm{Id} \in \mathrm{Gal}(L/K)$, which would contradict the characterization of $\mathrm{Gal}(K^{\mathrm{alg}}/M)$ (see remark after Theorem 2.1) .

According to [LM05], the p-Sylow subgroup $\mathrm{Aut}_k(\mathcal{C}_k)_1^{\#}$ of $\mathrm{Aut}_k(\mathcal{C}_k)^{\#}$ is an extra-special p-group of order pq^2. Moreover, one has :

$$0 \to \mathrm{Z}(\mathrm{Aut}_k(\mathcal{C}_k)_1^{\#}) \to \mathrm{Aut}_k(\mathcal{C}_k)_1^{\#} \to (\mathbb{Z}/p\mathbb{Z})^{2n} \to 0,$$

where $(\mathbb{Z}/p\mathbb{Z})^{2n}$ is identified with the group of translations $t \mapsto t+a$ extending to elements of $\mathrm{Aut}_k(\mathcal{C}_k)_1^{\#}$. Therefore we have morphisms

$$\mathrm{Gal}(M/K) \overset{i}{\hookrightarrow} \mathrm{Aut}_k(\mathcal{C}_k)_1^{\#} \overset{\varphi}{\to} \mathrm{Aut}_k(\mathcal{C}_k)_1^{\#}/\mathrm{Z}(\mathrm{Aut}_k(\mathcal{C}_k)_1^{\#}).$$

The composition is seen to be surjective since the image contains the q^2 translations $t \mapsto t + \overline{(y_i - y_1)/\lambda^{p/(1+q)}}$. Consequently, $i(\mathrm{Gal}(M/K))$ is a subgroup of $\mathrm{Aut}_k(\mathcal{C}_k)_1^{\#}$ of index at most p. So it contains $\Phi(\mathrm{Aut}_k(\mathcal{C}_k)_1^{\#}) = \mathrm{Z}(\mathrm{Aut}_k(\mathcal{C}_k)_1^{\#}) = \mathrm{Ker}\,\varphi$. It implies that i is an isomorphism. Thus $[M : K] = pq^2$. By **Step IV**, one has $M \subseteq K(y, f_{q,c}(y)^{1/p})$, hence $M = K(y, f_{q,c}(y)^{1/p})$.

We show, for later use, that $K(y_1)/K$ is Galois and that $\mathrm{Gal}(M/K(y_1)) = \mathrm{Z}(G)$. Indeed, $M/K(y_1)$ is p-cyclic and generated by σ defined by :

$$\sigma(y_1) = y_1 \text{ and } \sigma(f_{q,c}(y_1)^{1/p}) = \zeta_p^{-1} f_{q,c}(y_1)^{1/p}.$$

According to **Step IV**, σ acts on the stable model by :

$$\sigma(S) = S, \quad \sigma(\frac{Y}{f_{q,c}(y_1)^{1/p}}) = \frac{Y}{\zeta_p^{-1} f_{q,c}(y_1)^{1/p}} = \lambda\sigma(W) + 1 + SA_n(S).$$

Hence

$$\frac{\lambda W + 1 + SA_n(S)}{\zeta_p^{-1}} = \lambda\sigma(W) + 1 + SA_n(S),$$
$$\text{thus,} \quad \sigma(W) = \zeta_p W + 1 + SA_n(S).$$

It follows that, in reduction, σ induces the Artin-Schreier morphism that generates $\mathrm{Z}(\mathrm{Aut}_k(\mathcal{C}_k)_1^{\#})$. It implies that $K(y_1)/K$ is Galois, $\mathrm{Gal}(M/K(y_1)) = \mathrm{Z}(G)$ and $\mathrm{Gal}(K(y_1)/K) \simeq (\mathbb{Z}/p\mathbb{Z})^{2n}$.

We now prove the statements concerning the arithmetic of M/K. We assume that $c \in R$ with $v(c^p - c) \geq v(p)$ and we split the proof into 5 steps. Let y be a root of $L_c(X)$ and $b_n := (-1)(-p)^{1+p+\cdots+p^{n-1}}$. Note that $b_n^p = a_n$ and $L := K(y_1, \ldots, y_{q^2}) = K(y_1)$. We note that $v(c^p - c) \geq v(p)$, so

$v(\lambda^{p/(1+q)}) > v(c)$ implies $v(c) = 0$.

Step A : *The polynomial $L_c(X)$ is irreducible over K.*
One computes

$$\begin{aligned}(y^{q^2/p} - cb_n)^p &= y^{q^2} + (-c)^p a_n + \Sigma \\ &= a_n(1 + y^q(c+y))^{q-1}(c+y) + (-c)^p a_n + \Sigma \\ &= a_n \sum_{k=0}^{q-1} \binom{q-1}{k} y^{kq}(c+y)^{1+k} + (-c)^p a_n + \Sigma \\ &= a_n y + a_n(c + (-c)^p) + a_n \Sigma' + \Sigma,\end{aligned}$$

where $\Sigma := \sum_{k=1}^{p-1} \binom{p}{k} y^{kq^2/p}(-cb_n)^{p-k}$ and $\Sigma' := \sum_{k=1}^{q-1} \binom{q-1}{k} y^{kq}(c+y)^{1+k}$. Using **Step I** one checks that $v(\Sigma) > v(a_n y)$ and $v(\Sigma') \geq v(y^q) > v(y)$. Since $v(c^p - c) \geq v(p) > v(y)$, one gets :

$$v(y^{q^2/p} - cb_n) = \frac{v(a_n y)}{p},$$

and $t := p^{q^2}(y^{q^2/p} - cb_n)^{-(p-1)(q+1)} \in L$ has valuation $v_L(p)/q^2 = [L : \mathbb{Q}_p^{\mathrm{ur}}]/q^2$. So q^2 divides $[L : K]$. It implies that $L_c(X)$ is irreducible over K.

Step B : *Reduction step.*
The last non-trivial group G_{i_0} of the lower ramification filtration $(G_i)_{i \geq 0}$ of $G := \mathrm{Gal}(M/K)$ is a subgroup of $\mathrm{Z}(G)$ ([Ser79] IV §2 Corollary 2 of Proposition 9) and as $\mathrm{Z}(G) \simeq \mathbb{Z}/p\mathbb{Z}$, it follows that $G_{i_0} = \mathrm{Z}(G)$.

According to **Step VI** the group $H := \mathrm{Gal}(M/L)$ is $\mathrm{Z}(G)$. Consequently, the filtration $(G_i)_{i \geq 0}$ can be deduced from that of M/L and L/K (see [Ser79] IV §2 Proposition 2 and Corollary of Proposition 3).

Step C : *The ramification filtration of L/K is :*

$$(G/H)_0 = (G/H)_1 \supsetneq (G/H)_2 = \{1\}.$$

Since $K/\mathbb{Q}_p^{\mathrm{ur}}$ is tamely ramified of degree $(p-1)(q+1)$, one has $K = \mathbb{Q}_p^{\mathrm{ur}}(\pi_K)$ with $\pi_K^{(p-1)(q+1)} = p$ for some uniformizer π_K of K. In particular,

$$z := \frac{\pi_K^{q^2}}{y^{q^2/p} - cb_n},$$

is such that $t = z^{(p-1)(q+1)}$. Then, following the proof of **Step A**, z is a uniformizer of L. Let y and y' be two distinct roots of $L_c(X)$. Let $\sigma \in \mathrm{Gal}(L/K)$

such that $\sigma(y) = y'$. Then

$$\begin{aligned}\sigma(z) - z &= \frac{\pi_K^{q^2}}{y'^{q^2/p} - cb_n} - \frac{\pi_K^{q^2}}{y^{q^2/p} - cb_n} \\ &= \pi_K^{q^2} \frac{y^{q^2/p} - y'^{q^2/p}}{(y^{q^2/p} - cb_n)(y'^{q^2/p} - cb_n)},\end{aligned}$$

so $v(\sigma(z) - z) = 2v(z) - q^2 v(\pi_K) + v(y'^{q^2/p} - y^{q^2/p})$. It follows from :

$$(y - y')^{q^2/p} = y^{q^2/p} + (-y')^{q^2/p} + \sum_{k=1}^{\frac{q^2}{p}-1} \binom{q^2/p}{k} y^k (-y')^{\frac{q^2}{p}-k},$$

and $v(y) = v(y')$, $v(p) + \frac{q^2}{p} v(y) > \frac{q^2}{p} v(y - y')$ (use **Step I** and **Step V**) that $v(y^{q^2/p} - y'^{q^2/p}) = \frac{q^2}{p} v(y - y') = q^2 v(\pi_K)$. Hence $v(\sigma(z) - z) = 2v(z)$. This means that $(G/H)_2 = \{1\}$.

Step D : *Let $s := (q+1)(pq^2 - 1)$. There exist $u \in L$, $r \in \pi_L^s \mathfrak{m}$ such that*

$$f_{q,c}(y) u^p = 1 + p y^{q/p} (\frac{y^{q^2/p}}{b_n} - c) + r,$$

and $v_L(p y^{q/p} (\frac{y^{q^2/p}}{b_n} - c)) = s$.
To prove the second statement, we note that :

$$(\frac{y^{q^2/p}}{b_n})^p = f_{q,c}(y)^{q-1}(c + y) = \sum_{k=0}^{q-1} \binom{q-1}{k} y^{kq} (c+y)^{1+k} = c + y + \Sigma,$$

with $\Sigma := \sum_{k=1}^{q-1} \binom{q-1}{k} y^{kq} (c+y)^{1+k}$. We set $h := \frac{y^{q^2/p}}{b_n} - c$ and compute :

$$\begin{aligned}h^p &= (\frac{y^{q^2/p}}{b_n})^p + (-c)^p + \sum_{k=1}^{p-1} \binom{p}{k} (\frac{y^{q^2/p}}{b_n})^k (-c)^{p-k} \\ &= c + (-c)^p + y + \Sigma + \sum_{k=1}^{p-1} \binom{p}{k} (\frac{y^{q^2/p}}{b_n})^k (-c)^{p-k}.\end{aligned}$$

Since $v(c^p - c) \geq v(p) > v(y)$, $v(\Sigma) > v(y)$ and $v(\frac{y^{q^2/p}}{b_n}) \geq 0$, one gets $v_L(h) = v_L(y)/p = q^2 - 1$ and $v_L(p y^{q/p} h) = s$.

For the first claim, if $n \geq 2$ we choose :

$$u := 1 - cy^{q/p} + \sum_{k=0}^{n-2} \frac{y^{(1+q)p^k}}{(-p)^{1+p+\cdots+p^k}} = 1 + w.$$

Then, $f_{q,c}(y)u^p - 1 = 1 + cy^q + y^{1+q} + \Sigma_1 + cy^q\Sigma_1 + y^{1+q}\Sigma_1 - 1$ with :

$$\begin{aligned}\Sigma_1 &:= \sum_{k=1}^{p-1} \binom{p}{k} w^k + w^p = pw + \sum_{k=2}^{p-1} \binom{p}{k} w^k + w^p = pw + \Sigma' + w^p \\ &= p\left[-cy^{q/p} - \frac{y^{1+q}}{p} + \sum_{k=1}^{n-2} \frac{y^{(1+q)p^k}}{(-p)^{1+p+\cdots+p^k}}\right] + \Sigma' + w^p,\end{aligned}$$

and $\Sigma' := \sum_{k=2}^{p-1} \binom{p}{k} w^k$, where sums are eventually empty if $n = 2$ or $p = 2$. So :

$$\begin{aligned}f_{q,c}(y)u^p - 1 = cy^q - pcy^{q/p} + \sum_{k=1}^{n-2} \frac{py^{(1+q)p^k}}{(-p)^{1+p+\cdots+p^k}} + \Sigma' + w^p \\ + cpy^q w + cy^q\Sigma' + cy^q w^p + y^{1+q}pw + y^{1+q}\Sigma' + y^{1+q}w^p.\end{aligned}$$

Computation shows that $v(w) = v(y^{q/p})$ and we deduce that :

$$w^p = (-c)^p y^q + \sum_{k=0}^{n-2} \frac{y^{(1+q)p^{1+k}}}{(-p)^{p+\cdots+p^{1+k}}} \mod \pi_L^s \mathfrak{m}.$$

One checks that $v_L(y^q p) > s$, $v_L(y^q w^p) > s$ and $v_L(\Sigma') > s$. Hence :

$$\begin{aligned}f_{q,c}(y)u^p - 1 &= (c + (-c)^p)y^q - pcy^{q/p} + \sum_{k=1}^{n-2} \frac{py^{(1+q)p^k}}{(-p)^{1+\cdots+p^k}} + \sum_{k=0}^{n-2} \frac{y^{(1+q)p^{1+k}}}{(-p)^{p+\cdots+p^{1+k}}} \\ &= -pcy^{q/p} + \frac{y^{q/p(1+q)}}{(-p)^{p+\cdots+q/p}} = py^{q/p}\left(\frac{y^{q^2/p}}{b_n} - c\right) \mod \pi_L^s \mathfrak{m}.\end{aligned}$$

If $n = 1$, we choose $u := 1 - cy$ and check that the statement is still true.

Step E : *Computation of conductors.*
From **Step D**, one deduces that the extension M/L is defined by $X^p = 1 + phy^{q/p} + r$ with $r \in \pi_L^s \mathfrak{m}$. From lemma 2.1, one gets that $v_M(\mathcal{D}_{M/L}) = (p-1)(q+2)$. Hence

$$\mathbb{Z}/p\mathbb{Z} \simeq H_0 = H_1 = \cdots = H_{1+q} \supsetneq \{1\},$$

and according to **Step B** and **Step C** one has :

$$G = G_0 = G_1 \supsetneq \mathrm{Z}(G) = G_2 = \cdots = G_{1+q} \supsetneq \{1\}.$$

Let $\ell \neq p$ be a prime number. Since the G-modules $\mathrm{Jac}(C)[\ell]$ and $\mathrm{Jac}(\mathcal{C}_k)[\ell]$ are isomorphic (see [ST68] paragraph 2) one has that for $i \geq 0$:

$$\dim_{\mathbb{F}_\ell} \mathrm{Jac}(C)[\ell]^{G_i} = \dim_{\mathbb{F}_\ell} \mathrm{Jac}(\mathcal{C}_k)[\ell]^{G_i}.$$

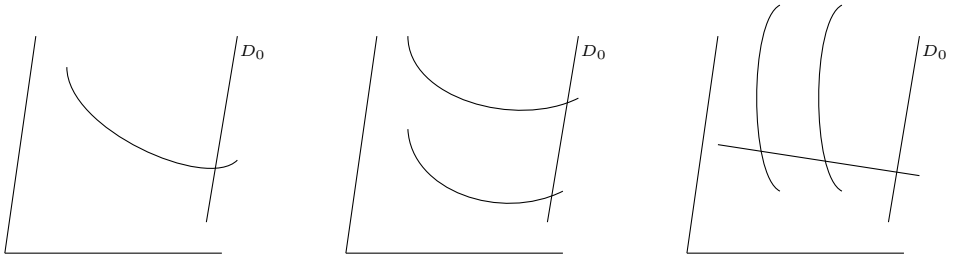
Moreover, for $0 \leq i \leq 1+q$ one has $\mathrm{Jac}(\mathcal{C}_k)[\ell]^{G_i} \subseteq \mathrm{Jac}(\mathcal{C}_k)[\ell]^{\mathrm{Z}(G)}$. Then, from $\mathcal{C}_k/\mathrm{Z}(G) \simeq \mathbb{P}^1_k$ (see end of **StepVI**) and lemma 2.2, it follows that for $0 \leq i \leq 1+q$, $\dim_{\mathbb{F}_\ell} \mathrm{Jac}(\mathcal{C}_k)[\ell]^{G_i} = 0$. Since $g(C) = q(p-1)/2$, one gets $f(\mathrm{Jac}(C)/K) = (2q+1)(p-1)$. Moreover, if $c \in \mathbb{Q}_p^{\mathrm{ur}}$, an easy computation shows that $\mathrm{sw}(\mathrm{Jac}(C)/\mathbb{Q}_p^{\mathrm{ur}}) = 1$. □

Remark : If $c \in R$ with $v(c) = (a/b)v(p) < v(\lambda^{p/(1+q)})$ and a and b both prime to p, then $L_c(X)$ is irreducible over K. Indeed, the expression of the valuation of any root y of $L_c(X)$ shows that the ramification index of $K(y)/K$ is q^2.

4 Monodromy of genus 2 hyperelliptic curves

We restrict to the case $p = 2$ and $\deg f(X) = 5$ of the introduction. In this situation, there are three types of geometry for the stable reduction (see Figure 1). For each type of degeneration, we will give an example of cover C/K with maximal wild monodromy and birationally given by $Y^2 = f(X) = 1 + b_2X^2 + b_3X^3 + b_4X^4 + X^5 \in R[X]$ over some R. Define $\mathcal{X}$ to be the R-model of C/K given by $Y^2 = f(X)$ and let's describe each degeneration type.

The Jacobian criterion shows that $\mathcal{X}_k/k$ has two singularities if and only if $\overline{b_3} \neq 0$. Type I occurs when $\mathcal{X}_k/k$ has two singularities and by blowing-up $\mathcal{X}$ at these points one obtains two elliptic curves. Type II (resp. type III) occurs in the one singularity case when there are two (resp. one) irreducible components of non 0 genus in the stable reduction. The elliptic curves E/k that we will encounter are birationally given by $w^2 - w = t^3$. They are such that $\mathrm{Aut}_k(E) \simeq \mathrm{Sl}_2(\mathbb{F}_3)$ has a unique 2-Sylow subgroup isomorphic to Q_8. We denote by D_0 the *original component* defined in Proposition 2.1.



Type III Type I Type II

Figure 1

Magma codes used in this section are available on the authors webpages.

Notations. For $f(X) \in R[X]$, let

$$f(X+x) = s_0(x) + s_1(x)X + s_2(x)X^2 + s_3(x)X^3 + s_4(x)X^4 + X^5,$$

be the Taylor expansion of f and define

$$T_f(Y) := s_1(Y)^2 - 4s_0(Y)s_2(Y).$$

Degeneration type III : This is the case of potential good reduction. For example, using notations of the previous section, let $K := \mathbb{Q}_2^{\mathrm{ur}}((-2)^{1/5})$ and C_1/K be the smooth, projective, geometrically integral curve birationally given by $Y^2 = 1+X^4+X^5 = f_{4,1}(X)$. Then, C_1/K has potential good reduction with maximal wild monodromy M/K, the group $\mathrm{Gal}(M/K)$ is an extra-special 2-group of order 2^5, $f(\mathrm{Jac}(C_1)/K) = 9$ and $\mathrm{sw}(\mathrm{Jac}(C_1)/\mathbb{Q}_2^{\mathrm{ur}}) = 1$.

Degeneration type I :

Proposition 4.1. *Let $\rho := 2^{2/3}$, $b_2, b_3, b_4 \in \mathbb{Q}_2^{\mathrm{alg}}$, $K := \mathbb{Q}_2^{\mathrm{ur}}(b_2, b_3, b_4)$ and C/K be the smooth, projective, geometrically integral curve birationally given by*

$$Y^2 = f(X) = 1 + b_2X^2 + b_3X^3 + b_4X^4 + X^5,$$

with $v(b_i) \geq 0$ and $v(b_3) = 0$. Assume that $1 + b_3b_2 + b_3^2b_4 \not\equiv 0 \mod \pi_K$. Then C has stable reduction of type I and

$$T_f(Y) = T_{1,f}(Y)T_{2,f}(Y) \text{ with } T_{i,f}(Y) \in K[Y],$$

such that $\overline{T_{1,f}}(Y) = Y^4 \in k[Y]$ and $\overline{T_{2,f}}(Y) = Y^4 + \overline{b_3}^2 \in k[Y]$. If $T_{1,f}(Y)$ and $T_{2,f}(Y)$ are irreducible over K and define linearly disjoint extensions, then C/K has maximal wild monodromy M/K with group $\mathrm{Gal}(M/K) \simeq Q_8 \times Q_8$.

Proof. Using Maple, one computes $T_f(Y)$ and reduces it mod 2. The statement about $T_f(Y)$ follows from Hensel's lemma.

Let y be a root of $T_f(Y)$. Define $\rho T = S = X - y$ and choose $s_0(y)^{1/2}$ and $s_2(y)^{1/2}$ such that $2s_0(y)^{1/2}s_2(y)^{1/2} = s_1(y)$. Then

$$\begin{aligned} f(S+y) &= s_0(y) + s_1(y)S + s_2(y)S^2 + s_3(y)S^3 + s_4(y)S^4 + S^5 \\ &= (s_0(y)^{1/2} + s_2(y)^{1/2}S)^2 + s_3(y)S^3 + s_4(y)S^4 + S^5 \\ &= (s_0(y)^{1/2} + s_2(y)^{1/2}\rho T)^2 + s_3(y)\rho^3 T^3 + s_4(y)\rho^4 T^4 + \rho^5 T^5. \end{aligned}$$

The change of variables

$$\rho T = S = X - y \ \text{ and } \ Y = 2W + (s_0(y)^{1/2} + s_2(y)^{1/2}S),$$

induces

$$W^2 + (s_0(y)^{1/2} + s_2(y)^{1/2}S)W = s_3(y)T^3 + s_4(y)\rho T^4 + \rho^2 T^5,$$

which is an equation of a quasi-projective flat scheme over $K(y, f(y)^{1/2})^\circ$ with special fiber given by $w^2 - w = t^3$.

Let $(y_i)_{i=1,\dots,4}$ (resp. $(y_i)_{i=5,\dots,8}$) be the roots of $T_{1,f}(Y)$ (resp. $T_{2,f}(Y)$). Then, for any $i \in \{1,\dots,4\}$ and $j \in \{5,\dots,8\}$, the above computations show that C has stable reduction over $L := K(y_i, y_j, f(y_i)^{1/2}, f(y_j)^{1/2})$ (use [Liu02] 10.3.44). Moreover two distinct roots of $T_{1,f}(Y)$ (resp. $T_{2,f}(Y)$) induce equivalent Gauss valuations on $K(C)$, else it would contradict the uniqueness of the stable model. In particular,

$$\begin{aligned} v(y_i - y_j) \geq v(\rho),\ i \neq j \in \{1,2,3,4\} \ \text{ or } \ i \neq j \in \{5,6,7,8\}, \\ v(y_i - y_j) = 0,\ i \in \{1,2,3,4\} \ \text{ and } \ j \in \{5,6,7,8\}, \end{aligned} \tag{7}$$

which implies that $v(\mathrm{disc}(T_{1,f}(Y))) + v(\mathrm{disc}(T_{2,f}(Y))) = v(\mathrm{disc}(T_f(Y)))$ and $\mathrm{disc}(T_{i,f}(Y)) \geq 12v(\rho) = 8v(2)$ for $i = 1, 2$. These are equalities if and only if (7) are all equalities. Using Maple, one has

$$2^{-16}\,\mathrm{disc}(T_f(Y)) = b_3^8(1 + b_3b_2 + b_3^2b_4)^4 \mod 2,$$

thus one gets that (7) are all equalities, therefore

$$0, \frac{y_2 - y_1}{\rho}, \frac{y_3 - y_1}{\rho}, \frac{y_4 - y_1}{\rho},$$

are all distinct mod π_K. Applying Hensel's lemma to $T_{1,f}(\rho Y + y_1)$ shows that $K(y_1)/K$ is Galois. The same holds for $K(y_5)/K$.

Let's denote by E_1/k and E_2/k (resp. ∞_1 and ∞_2) the genus 1 curves in the stable reduction of C (resp. their crossing points with D_0). The group $\mathrm{Aut}_k(\mathcal{C}_k)^{\#} \simeq \mathrm{Aut}_{k,\infty_1}(E_1) \times \mathrm{Aut}_{k,\infty_2}(E_2)$ has a unique 2-Sylow subgroup isomorphic to $\mathrm{Syl}_2(\mathrm{Aut}_{k,\infty_1}(E_1)) \times \mathrm{Syl}_2(\mathrm{Aut}_{k,\infty_2}(E_2))$ where $\mathrm{Aut}_{k,\infty_i}(E_i) \simeq \mathrm{Sl}_2(\mathbb{F}_3)$ denotes the subgroup of $\mathrm{Aut}_k(E_i)$ leaving ∞_i fixed.

First, we show that L/K is the monodromy extension M/K of C/K. Let $\sigma \in \mathrm{Gal}(L/K)$ inducing the identity on $\mathcal{C}_k/k$. We show that $\forall i \in \{1,\dots,8\}$, $\sigma(y_i) = y_i$. Otherwise, since σ is an isometry, $\sigma(y_1) \notin \{y_5,\dots,y_8\}$ and we can assume that $\sigma(y_1) = y_2$. So :

$$\sigma(\frac{X-y_1}{\rho}) = \frac{X-y_1}{\rho} + \frac{y_1-y_2}{\rho},$$

so σ does not induce the identity on $\mathcal{C}_k/k$. If $\sigma(s_0(y_i)^{1/2}) = -s_0(y_i)^{1/2}$ for some i then $\sigma(s_2(y_i)^{1/2}) = -s_2(y_i)^{1/2}$ and

$$\sigma(W) - W = s_0(y_i)^{1/2} + s_2(y_i)^{1/2}\rho T,$$

therefore σ acts non trivially on $\mathcal{C}_k/k$. It implies that for all $i \in \{1,\dots,8\}$, $\sigma(s_0(y_i)^{1/2}) = s_0(y_i)^{1/2}$ and $\sigma = \mathrm{Id}$. Since $M \subseteq L$, this shows that $L = M$.

Now, we show that the wild monodromy is maximal assuming that $T_{1,f}(Y)$ and $T_{2,f}(Y)$ are irreducible over K and define linearly disjoint extensions. One has natural morphisms :

$$\mathrm{Gal}(M/K) \overset{i}{\hookrightarrow} Q_8 \times Q_8 \overset{p}{\longrightarrow} Q_8 \times Q_8/\mathrm{Z}(Q_8) \overset{q}{\longrightarrow} Q_8/\mathrm{Z}(Q_8) \times Q_8/\mathrm{Z}(Q_8).$$

For any $i \in \{1,\dots,4\}$ and $j \in \{5,\dots,8\}$, $(i,j) \neq (1,5)$, there exists $\sigma_{i,j} \in \mathrm{Gal}(M/K)$ such that :

$$\sigma_{i,j}(y_1) = y_i \ \text{ and } \ \sigma_{i,j}(y_5) = y_j,$$

which is seen to act non trivially on $\mathcal{C}_k/k$. The composition $q \circ p \circ i$ is then surjective, it implies that $p \circ i(\mathrm{Gal}(M/K))$ is a subgroup of $Q_8 \times Q_8/\mathrm{Z}(Q_8)$ of index at most 2 so it contains $\Phi(Q_8 \times Q_8/\mathrm{Z}(Q_8)) = \mathrm{Z}(Q_8) \times \{1\} = \mathrm{Ker}\, q$. It follows that $p \circ i$ is onto and $i(\mathrm{Gal}(M/K))$ is a subgroup of $Q_8 \times Q_8$ of index at most 2 so it contains $\Phi(Q_8 \times Q_8) = \mathrm{Z}(Q_8) \times \mathrm{Z}(Q_8) \supseteq \mathrm{Ker}\, p$. Finally, one has $i(\mathrm{Gal}(M/K)) = Q_8 \times Q_8$. □

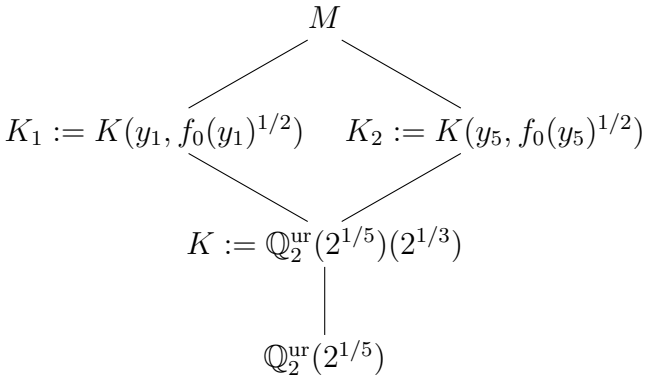
Example : Let $f_0(X) := 1 + 2^{3/5}X^2 + X^3 + 2^{2/5}X^4 + X^5$ and $K := \mathbb{Q}_2^{\mathrm{ur}}(2^{\frac{1}{15}})$. Then, one checks using Magma that $T_{f_0}(Y) = T_{1,f_0}(Y)T_{2,f_0}(Y)$ where $T_{1,f_0}(Y)$ and $T_{2,f_0}(Y)$ are irreducible polynomials over K and $T_{2,f_0}(Y)$ is irreducible over the decomposition field of $T_{1,f_0}(Y)$. So, the curve C_0/K defined by $Y^2 = f_0(X)$ has maximal wild monodromy M/K with group $\mathrm{Gal}(M/K) \simeq Q_8 \times Q_8$.

```
q2:= pAdicField(2,8);
q2x<x>:=PolynomialRing(q2);
k<pi>:=TotallyRamifiedExtension(q2,x^15-2);
K<rho>:=UnramifiedExtension(k,8);
Ky<y>:=PolynomialRing(K);
b3:=1;
b2:=pi^9;
b4:=pi^6;
T:=(2*b2*y+3*b3*y^2+4*b4*y^3+5*y^4)^2-
4*(1+b2*y^2+b3*y^3+b4*y^4+y^5)*(b2+3*b3*y+6*b4*y^2+10*y^3);
F,a,A:=Factorization(T: Extensions:= true);
Degree(F[1][1]);Degree(F[2][1]);
L1:=A[1]`Extension;
L1Y<Y>:=PolynomialAlgebra(L1);
TY:=L1Y!Eltseq(T);
G:=Factorization(TY);
G[1][2];G[2][2];G[3][2];G[4][2];G[5][2];
Degree(G[5][1]);
```

This has the following consequence for the Inverse Galois Problem :

Corollary 4.1. *With the notations of the above example, let $(y_i)_{i=1,\dots,8}$ be the roots of $T_{f_0}(Y)$ and $M = K(y_1, \dots, y_8, f_0(y_1)^{1/2}, \dots, f_0(y_8)^{1/2})$. Then M/K is Galois with Galois group isomorphic to $Q_8 \times Q_8 \simeq \mathrm{Syl}_2(\mathrm{Aut}_k(\mathcal{C}_k)^{\#})$.*

We now give results about the arithmetic of the monodromy extension of the previous example.



First of all, $M/\mathbb{Q}_2^{\mathrm{ur}}(2^{1/5})$ is the monodromy extension of $C_0/\mathbb{Q}_2^{\mathrm{ur}}(2^{1/5})$. Indeed, let θ be a primitive cube root of unity. The curve C_0 has a stable

model over M and $\sigma \in \mathrm{Gal}(K/\mathbb{Q}_2^{\mathrm{ur}}(2^{1/5}))$ defined by $\sigma(2^{1/3}) = \theta 2^{1/3}$ acts non trivially on the stable reduction by $t \mapsto \bar{\theta} t$. It implies that

$$\mathrm{Gal}(M/\mathbb{Q}_2^{\mathrm{ur}}(2^{1/5})) \hookrightarrow \mathrm{Sl}_2(\mathbb{F}_3) \times \mathrm{Sl}_2(\mathbb{F}_3).$$

Moreover, M/K_1 is Galois with group isomorphic to Q_8. Indeed, from the proof of proposition 4.1, since $\mathrm{Gal}(M/K_1)$ acts trivially on one of the two elliptic curves of the stable reduction, one has the injection :

$$\mathrm{Gal}(M/K_1) \hookrightarrow Q_8 \times \{\mathrm{Id}\}.$$

Moreover the image of this injection is mapped onto $Q_8/\mathrm{Z}(Q_8)$ so $\mathrm{Gal}(M/K_1) \simeq Q_8$. The extensions K_1/K and K_2/K being linearly disjoint. It implies that $\mathrm{Gal}(K_2/K) \simeq Q_8$ and it follows that $\mathrm{Gal}(K_2/\mathbb{Q}_2^{\mathrm{ur}}(2^{1/5})) \simeq \mathrm{Sl}_2(\mathbb{F}_3)$. Similarly, $\mathrm{Gal}(K_1/\mathbb{Q}_2^{\mathrm{ur}}(2^{1/5})) \simeq \mathrm{Sl}_2(\mathbb{F}_3)$.

This has consequences on the possible ramification subgroups arising in the filtrations of ${}_1G := \mathrm{Gal}(K_1/K)$ and ${}_2G := \mathrm{Gal}(K_2/K)$. Namely there are no subgroups of order 4, otherwise there would be a normal subgroup of order 4 in $\mathrm{Sl}_2(\mathbb{F}_3)$. So the possible subgroups arising in the ramification filtrations of ${}_1G$ and ${}_2G$ are Q_8, $\mathrm{Z}(Q_8)$ and $\{1\}$.

Using Magma one computes the lower ramification filtrations

$$\begin{aligned} {}_1G = ({}_1G)_0 = ({}_1G)_1 \supsetneq \mathrm{Z}({}_1G) = ({}_1G)_2 = ({}_1G)_3 \supsetneq \{1\}, \\ {}_2G = ({}_2G)_0 = \cdots = ({}_2G)_5 \supsetneq \mathrm{Z}({}_2G) = ({}_2G)_6 = \cdots = ({}_2G)_{69} \supsetneq \{1\}. \end{aligned}$$

In order to compute the lower ramification filtration of $\mathrm{Gal}(M/K)$, we now determine its upper ramification filtration since it enjoys peculiar arithmetic properties. Using lemma 3.5 of [Kid03] and the expressions of $\varphi_{K_1/K}$ and $\varphi_{K_2/K}$, one sees that K_1/K and K_2/K are *arithmetically disjoint*. According to [Yam68] theorem 3, one has for any $u \in \mathbb{R}$:

$$\mathrm{Gal}(M/K)^u \simeq {}_1G^u \times {}_2G^u.$$

So one gets :

$$\mathrm{Gal}(M/K)^u \simeq \begin{cases} {}_1G \times {}_2G, & -1 \leq u \leq 1, \\ \mathrm{Z}({}_1G) \times {}_2G, & 1 < u \leq 3/2, \\ \{1\} \times {}_2G, & 3/2 < u \leq 5, \\ \{1\} \times \mathrm{Z}({}_2G), & 5 < u \leq 21, \\ \{1\} \times \{1\}, & 21 < u. \end{cases}$$

One deduces the lower ramification filtration of $\mathrm{Gal}(M/K)$:

$$\mathrm{Gal}(M/K)_i \simeq \begin{cases} {}_1G \times {}_2G, & -1 \leq i \leq 1, \\ \mathrm{Z}({}_1G) \times {}_2G, & 2 \leq i \leq 3, \\ \{1\} \times {}_2G, & 4 \leq i \leq 31, \\ \{1\} \times \mathrm{Z}({}_2G), & 32 \leq i \leq 543, \\ \{1\} \times \{1\}, & 544 \leq i. \end{cases}$$

Let denote the genus 1 irreducible components of $\mathcal{C}_k/k$ by E_1 and E_2. Let H_1 (resp. H_2) be a finite subgroup of $\mathrm{Syl}_2(\mathrm{Aut}_{k,\infty_1}(E_1))$ (resp. $\mathrm{Syl}_2(\mathrm{Aut}_{k,\infty_2}(E_2))$) and $\ell \neq 2$ be a prime number. One has :

$$\mathrm{Pic}^\circ(\mathcal{C}_k)[\ell]^{H_1\times H_2} = \mathrm{Pic}^\circ(E_1)[\ell]^{H_1} \times \mathrm{Pic}^\circ(E_2)[\ell]^{H_2}.$$

According to lemma 2.2 one has $\dim_{\mathbb{F}_\ell} \mathrm{Pic}^\circ(E_i)[\ell]^{H_i} = 2g(E_i/H_i)$. It follows that $\mathrm{sw}(\mathrm{Jac}(C_0)/K) = 45$.

Degeneration type II :

Proposition 4.2. *Let $a^9 = 2$, $K := \mathbb{Q}_2^{\mathrm{ur}}(a)$, $\rho := a^4$ and C/K be the smooth, projective, geometrically integral curve birationally given by*

$$Y^2 = f(X) = 1 + a^3X^2 + a^6X^3 + X^5.$$

Then, C has stable reduction of type II and C/K has maximal wild monodromy M/K with group $\mathrm{Gal}(M/K) \simeq (Q_8 \times Q_8) \rtimes \mathbb{Z}/2\mathbb{Z}$.

Proof. Using Magma, one determines the Newton polygon of $T_f(Y)$. Then, $T_f(Y)$ has 8 roots $(y_i)_{i=1,\dots,8}$ of valuation $\frac{7}{24}v(2)$. By considering the Newton polygon of $\Delta(Z) = (T_f(Z+y_1) - T_f(y_1))/Z$, one shows that $\Delta(Z)$ has 3 roots (say $y_2 - y_1$, $y_3 - y_1$ and $y_4 - y_1$) of valuation $v(\rho)$ and 4 roots of valuation $v(2)/3$.

Let y be a root of $T_f(Y)$. Define $\rho T = S = X - y$ and choose $s_0(y)^{1/2}$ and $s_2(y)^{1/2}$ such that $2s_0(y)^{1/2}s_2(y)^{1/2} = s_1(y)$. Then the change of variables

$$\rho T = S = X - y \ \text{ and } \ Y = 2W + (s_0(y)^{1/2} + s_2(y)^{1/2}S),$$

induces

$$W^2 + (s_0(y)^{1/2} + s_2(y)^{1/2}S)W = \frac{s_3(y)\rho^3}{4}T^3 + \frac{s_4(y)\rho^4}{4}T^4 + \frac{\rho^5}{4}T^5,$$

which is an equation of a quasi-projective flat scheme over $K(y, f(y)^{1/2})$ with special fiber given by $w^2 - w = t^3$. The same argument as in the degeneration type I shows that C has stable reduction of type II over $L = K(y_1, \dots, y_8, f(y_1)^{1/2}, \dots, f(y_8)^{1/2})$.

We first show that L/K is the monodromy extension M/K of C/K. Let $\sigma \in \mathrm{Gal}(L/K)$ inducing the identity on $\mathcal{C}_k/k$. We show that $\forall i \in \{1, \dots, 8\}$, $\sigma(y_i) = y_i$. Else, for example, $\sigma(y_1) = y_2$ or $\sigma(y_1) = y_5$. It follows from the properties of the roots of $\Delta(Z)$ that, if $\sigma(y_1) = y_2$ then σ acts by non trivial translation on $\mathcal{C}_k/k$ and if $\sigma(y_1) = y_5$ then σ acts on $\mathcal{C}_k/k$ by permuting the

genus 1 components. Once again, the same computations as in the degeneration type I show that $\forall i \in \{1,\dots,8\}$, $\sigma(f(y_i)^{1/2}) = f(y_i)^{1/2}$. Since $M \subseteq L$, one gets $M = L$.

Now, we show that the wild monodromy is maximal. Let's consider the canonical morphism :

$$\mathrm{Gal}(M/K) \overset{i}{\hookrightarrow} \mathrm{Syl}_2(\mathrm{Aut}_k(\mathcal{C}_k)^{\#}) \simeq (Q_8 \times Q_8) \rtimes \mathbb{Z}/2\mathbb{Z}.$$

One sees $Q_8 \times Q_8$ as the subgroup $(Q_8 \times Q_8) \rtimes \{1\}$ of $(Q_8 \times Q_8) \rtimes \mathbb{Z}/2\mathbb{Z}$. Set $H := i(\mathrm{Gal}(M/K)) \cap (Q_8 \times Q_8)$. One has natural morphisms :

$$H \overset{p}{\longrightarrow} Q_8 \times Q_8/\mathrm{Z}(Q_8) \overset{q}{\longrightarrow} Q_8/\mathrm{Z}(Q_8) \times Q_8/\mathrm{Z}(Q_8).$$

Using Magma one shows that $T_f(Y)$ is irreducible over K and over $K(y_1)$ one has the following decomposition in irreducible factors :

$$T_f(Y) = \prod_{i=1}^{4}(Y - y_i)T_2(Y),$$

and $T_2(Y)$ decomposes over $K(y_1, y_5)$. It implies that $q \circ p \circ i$ is surjective and $p(H)$ is a subgroup of index at most 2 so it contains $\Phi(Q_8 \times Q_8/\mathrm{Z}(Q_8))$ and as for type I, one has $p(H) = Q_8 \times Q_8/\mathrm{Z}(Q_8)$. It implies that H is a subgroup of $Q_8 \times Q_8$ of index at most 2 and again $H = Q_8 \times Q_8$, that is $Q_8 \times Q_8 \subseteq i(\mathrm{Gal}(M/K))$. Finally on has a natural morphism :

$$(Q_8 \times Q_8) \rtimes \mathbb{Z}/2\mathbb{Z} \overset{r}{\longrightarrow} (Q_8/\mathrm{Z}(Q_8) \times Q_8/\mathrm{Z}(Q_8)) \rtimes \mathbb{Z}/2\mathbb{Z}.$$

The composition $r \circ i$ is surjective since there exist $\sigma, \tau \in \mathrm{Gal}(M/K)$ such that for $i \in \{1,\dots,4\}$ and $j \in \{5,\dots,8\}$

$$\begin{aligned} &\sigma(y_1) = y_i \ \text{ and } \ \sigma(y_5) = y_j, \\ &\tau(y_1) = y_5. \end{aligned}$$

Since the index of $i(\mathrm{Gal}(M/K))$ in $(Q_8 \times Q_8) \rtimes \mathbb{Z}/2\mathbb{Z}$ is at most 2, this group contains $\Phi((Q_8 \times Q_8) \rtimes \mathbb{Z}/2\mathbb{Z}) \supseteq \mathrm{Ker}\, r$ so $i(\mathrm{Gal}(M/K)) = (Q_8 \times Q_8) \rtimes \mathbb{Z}/2\mathbb{Z}$. □

Again we derive the following result for the Inverse Galois Problem :

Corollary 4.2. *With the notations of Proposition 4.2, let $(y_i)_{i=1,\dots,8}$ be the roots of $T_f(Y)$ and $M = K(y_1,\dots,y_8, f(y_1)^{1/2},\dots,f(y_8)^{1/2})$. Then M/K is Galois with Galois group isomorphic to $(Q_8 \times Q_8) \rtimes \mathbb{Z}/2\mathbb{Z} \simeq \mathrm{Syl}_2(\mathrm{Aut}_k(\mathcal{C}_k)^{\#})$.*

Remark. Throughout this paper, we have described monodromy extensions as decomposition fields of explicit polynomials being p-adic approximations of the so called *monodromy polynomial* of [LM06]. The point is that the roots of the monodromy polynomial are the centers of the blowing-ups giving the stable reduction of a p-cyclic cover of $\mathbb{P}^1_K$ with equidistant geometry. For a given genus, the expression of the monodromy polynomial is somehow generic, making it quite complicated. Since p-adically close polynomials with same degrees define the same extensions, it was natural to drop terms having a small p-adic contribution in our examples to obtain modified monodromy polynomials easier to handle than the actual monodromy polynomial.

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