A posteriori error estimator framework for PDEs

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Background

1. Master Applied Mathematics, Université de Bordeaux 1, June 2003
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Impact of numerical simulation

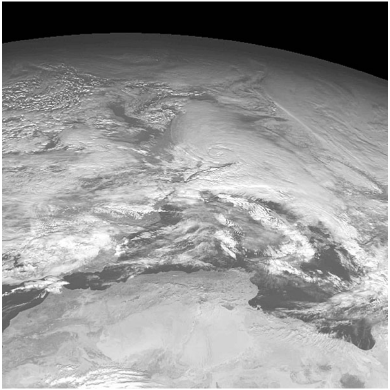


Figure: 1999 : Storm system Lothar over Europe

Interrogations

- Simulation is a bridge between theory and experiments DOE report from D. Keyes.
- How to make them reliable ?
- How reliable decision can be based on the outcome of a software expressing a mathematical model ?
- These issues raise the concept of Solution Validation and Verification.

Concept of solution verification

Definition

Quantitative evaluation of the numerical error of a given solution to the PDEs (ref. Oberkampf, Trucano)

- Estimate the accuracy of a given solution is the primary goal of solution verification
- It is often impossible to perform a complete and rigorous analysis for complex PDEs.
- The problem can be addressed by
  1. Explicit discretization robustness and convergence studies
  2. Formal error estimation procedures
  3. Inference from test problem suites and from previous experience
- Numerical error estimation is strongly dependent on the quality and completeness of code verification.

Concept of A posteriori error estimates

Definition

From word-net : **A posteriori : involving reasoning from facts or particulars to general principals or from effects to causes**.

Consequently, A Posteriori error estimates make use of

- A priori information
- Computational results from a previous numerical solution using the same numerical algorithm on the same PDE and initial and boundary data.
- Information extracted can be estimates or convergence characteristics.

Examples of A Posteriori estimates

- AIAA Guide for the Verification and Validation of Computational Fluid Dynamics Simulations.
- Finite Elements
  - ZZ recovery method - see Zienkiewicz et Al, and ref.
  - Equilibrated residual method for FE .- see Ainsworth & Oden and ref.
  - A posteriori Finite-Element free constant output bounds - see Patera and ref.
- Extrapolation Based methods
  - Richardson Extrapolation (h-extrapolation)
  - Order Extrapolation (p-extrapolation)
- Stochastic method in the Bayesian framework - ref Glimm et Al..

Facts on Richardson extrapolation

- It is a popular method in Computational Fluid Dynamics (**CFD**) because of its straightforward implementation that is code (and " PDE ") independent.
- It uses a sequence of meshes with distinct refinement to estimate the spatial discretization error.
- Can be extend to temporal discretization.
- Can be apply to large variety of discretization method

Overview of RE

- Let E be a normed linear space, $|| \, ||$ its norm, $v \in E$, $p > 0$, and $h \in (0, h_0)$. $u^i \in E, \; i = 1..3$ have the following asymptotic expansion,

$$u^i \;=\; v + C(\frac{h}{2^{i-1}})^p + \delta,$$

  with C positive constant independent of h, and $||\delta|| = o(h^p)$.
- For known p, Richardson extrapolation formula,

$$v_r^i = \frac{2^p \; u^{i+1} - u^i}{2^p - 1}, \;\; i = 1, 2$$

- Provides improved convergence:

$$||v - v_r^i|| = o(h^p)$$

Potential pitfalls

- Are the (3D) meshes fine enough to satisfies accurately the a priori convergence estimates that are only asymptotic relations in nature?
- What can be done, if the order of convergence of a PDE code is space dependent and eventually physical parameter's dependent?
- Can we afford three grid levels with a coarse grid solution that has a satisfactory level of accuracy, to be used in RE?
- Can we use RE to provide a posteriori error estimates?
- Richardson's method produces different estimates of error and uses different norms

Problem

A code that provides a set of discrete approximations of a (set of) PDE(s) for example Navier Stokes equations or Heat transfer equations.

- Provided that one can obtain the definition of the residual of the PDE approximation, the existence of a stability estimate on the approximation of the PDE's problem and two grid solutions, *find automatically the order of convergence*
- Using two or three different grid solutions (not necessarily with uniformly increasing mesh resolution), *obtain a solution with improved accuracy*
- Derive reliable a posteriori error bounds from coarse grid approximation of complex PDE problems.

Solution Procedure

- Simple to implement and works with a code independent from the main code procedure.
- With arithmetic cost negligible compare to a direct computation of a very fine grid solution.
- A general tool that can be applied to variational, FV or FD formulations, with irregular meshes, non linearities etc...
- Able to enhance the numerical accuracy and efficiency of simulation with complex physical model and trust in the context of code verification.
- Able to increase the overall numerical efficiency of the solution procedure when combined to multilevel procedure.

Problem

- This concept was introduced by Garbey and Shyy in 2002.
- Boundary value problem (Ω is a polygonal domain and $n = 2$ or 3) :

$$L[u(x)] = f(x), x \in \Omega \subset \mathbb{R}^n, u = g \quad \text{on} \quad \partial\Omega.$$

- Assume that the PDE problem is well posed and has a unique solution.
- We consider an approximation on a family of meshes $M(h)$ parametrized by $h > 0$ a small parameter.
- We denote symbolically the corresponding family of linear systems

$$A_h U_h = F_h.$$

- Let p_h denotes the projection of the continuous solution u onto the mesh $M(h)$. We assume a priori that ($||.||$ is a given discrete norm):

$$||U_h - p_h(u)|| \to 0, \quad \text{as} \quad h \to 0,$$

Optimized Extrapolation Technique

- Let $M(h_1)$ and $M(h_2)$ be two $\neq$ meshes used to build two approximations U_1 and U_2 of the PDE problem.
- A consistent linear extrapolation writes

$$\alpha U_1 + (1-\alpha)U_2,$$

  where α is a weight function.
- In classical Richardson Extrapolation (RE) α is a constant.
- In our optimized extrapolation method α is a function solution of the following optimization problem:

  P_α: *Find* $\alpha \in \Lambda(\Omega) \subset L_\infty$ *such that* $G(\alpha U_1 + (1-\alpha)U_2)$ *is minimum.*
- For computational efficiency, $\Lambda(\Omega)$ should be a finite vector space of very small dimension compared to the dimension of A_h.

General Idea

- One can choose to work with a posteriori FE error estimates:
- **From now on, and to make our technique general, we will work with discrete value functions and discrete norms: Why is it possible?**
- Our ambition: a numerical estimate on $||U_j - U_\infty||, j = 1, 2,$ without computing U_∞.
- $M(h_\infty)$ should capture a priori all the scales needed.
- In practice $h_\infty << h_1, h_2$.
- The solution U_j can be verified, assuming convergence of the approximation method, i.e $U_\infty \rightarrow u$, as $h_\infty \rightarrow 0$.
- Reuse extensive knowledge of Physics and Asymptotic Analysis.
- Reuse Stability Theory from Linear Algebra.

Practical Consequences

- Both coarse grid solutions U_1 and U_2 must be projected onto $M(h_\infty)$.
- The objective function is a discrete norm of the residual:

$$G(U^\alpha) = ||A_{h_\infty} U^\alpha - F_{h_\infty}||, \text{where} U^\alpha = \alpha \tilde{U}_1 + (1-\alpha)\tilde{U}_2$$

  The Optimized Extrapolated Solution (OES) if it exists, is denoted $V_e = \alpha_e U_1 + (1-\alpha_e)U_2$.
- The choice of the discrete norm depends on the property of the solution.
- One can choose to work in a subspace:
  - Estimate on a functional of the solution.
  - Estimate in sub-domain.

$$div(\rho\nabla u) = f$$

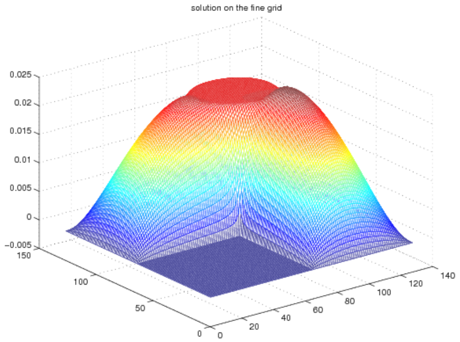


Figure: Stiff poisson problem on a fine grid (Garbey and Shyy)

- $\rho \approx 100$ in the disc, **one** elsewhere.
- Domain has a L-shape.
- coarse grid solutions: $h_1 = 1/14, h_2 = 1/20, h_3 = 1/26.$
- fine grid: $h^0 = 1/128.$

Error Estimate in L_2 norm

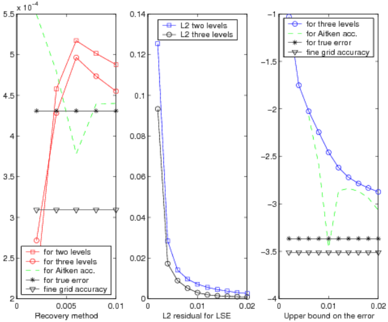


Figure: Error estimation in L_2 norm (Garbey and Shyy)

Problem setup I

- Assumption: well posed parabolic problem with a unique solution.

$$\frac{\partial u}{\partial t} = N[u], (x,t) \in \Omega \times (0,T), \quad (1)$$
$$u_{|\partial\Omega} = g(t), t \in (0,T), \quad (2)$$
$$u(x,0) = v(x), x \in \Omega. \quad (3)$$

- The coarse grid solution used in the numerical solution corresponding to the discretization (h,dt), $(h/2,dt)$, $(h,dt/2)$,$(h/2,dt/2)$ are $v^j_{dx,dt}$, $j = 1\ldots3$
- The fine grid $M(h_\infty)$ used in OES corresponds to $(h/4,dt/4)$.
- The coarse grid solutions $v^j_{dx,dt}$ are projected on the fine grid with second (or third order) accuracy. We denote them by $\tilde{U}_{dx,dt}$.

OES Formulation

- Find the three weight functions $\alpha_j, j = 1..3$ such that the residual
$$\sum_{j=1..4} \alpha_j \tilde{U}^{n+1}_{dx,dt} - H(\sum_{j=1..4} \alpha_j \tilde{U}^{n}_{dx,dt}),$$
is minimum in the discrete norm on the space time grid
$$\{idx\}_{i=1...N} \times \{t^n, t^n + dt, t^n + 2dt, t^n + 3dt, t^{n+1}\}$$
where H is some objective function
- The asymptotic expanstion writes
$$U_{dx,dt} - u = C_1 dx^{p_x} + C_2 dt^{p_t} + O(dx^{q_x}, dt^{q_t}), \tag{4}$$

Theorem on continuous functions

Theorem

If $\alpha_j \in C^0(\Omega_x \times \Omega_t), j = 1 \dots 3$ *, and* $v^j_{dx,dt} - v^4_{dx,dt} = O(dx^{p_x}, dt^{p_t})$ *then there exists* M *such that*

$$u = \sum_{j=1}^{4} \alpha_j^M v^j_{dx,dt} + O(dx^{p_x}, dt^{p_t}) \times O(M^{-1}) \qquad (5)$$

- In practice only an approximation to order $\varepsilon = M^{-1}$ is needed to compute α_i.

Richardson Extrapolation in space and time

Theorem

There exists a unique linear combination of the coarse grid solutions $U_{i,n}$ with constant weights $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ such that

$$\alpha_1 U_{1,1} + \alpha_2 U_{2,1} + \alpha_3 U_{1,2} + \alpha_4 U_{2,2} - u = O(dx^{p_x}) + O(dt^{p_t}), \qquad (6)$$

The $(\alpha_i)_{i=1\ldots3}$ are:

$$\alpha_1 = \frac{1}{(2^{p_x}-1)(2^{p_t}-1)}, \quad \alpha_2 = -\frac{2^{p_t}}{(2^{p_x}-1)(2^{p_t}-1)},$$
$$\alpha_3 = -\frac{2^{p_x}}{(2^{p_x}-1)(2^{p_t}-1)}$$

Further, the consistency of the extrapolation formula implies

$$\alpha_4 = 1 - \alpha_1 - \alpha_2 - \alpha_3.$$

On the precision of OES

Theorem

If the following two assumptions are true

- *the asymptotic expansion is valid in the discrete L^2 norm for the coarse grid solution used in OES,*
- *the consistency error for the one-step scheme is asymptotically equivalent to the error on the solution*

then the OES solution for the α_j coefficients is asymptotically equivalent to the RE solution within order 2.

Problem Definition

- **Thermal wave problem similar to Ropp et Al in JCP 2004,**

$$\frac{\partial T}{\partial t} = \Delta T - 2T(T-1)(2T-1).$$

- Benchmark problem exhibits a traveling wave

$$T(x,y,t) = 1 - \tanh(x+y-2t)$$

with wave speed is of order one.

- Experiment with constant extrapolation coefficient.
- Post-processing of fine grid solution by few SSOR.
- We use the unconstrained minimization subroutine of matlab, to compare results with different choices of the norm, i.e either discrete L_2 norm or maximum norm.
- We have three unknown coefficients and start the search from the set of RE coefficients.

Solution

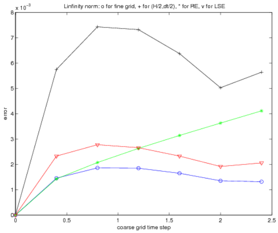


Figure: Evolution of the residual of in time with a Crank Nicholson scheme

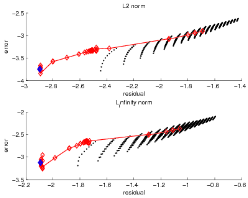


Figure: Optimization path from Richardson Extrapolation in red to the LSE optimum solution in blue for a Crank-Nicholson scheme

Problem Definition

- **The model we are using is the one proposed by Majda**

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}[F(u) - q_0 Z] = \varepsilon \frac{\partial^2 u}{\partial x^2} \quad (7)$$

$$Z_x = \varepsilon^{-1}\phi(u)Z \quad (8)$$

- Experiment with constant extrapolation coefficient.
- Post-processing of fine grid solution by few SSOR.
- We use the unconstrained minimization subroutine of matlab, to compare results with different choices of the norm, i.e either discrete L_2 norm or maximum norm.
- We have three unknown coefficients and start the search from the set of RE coefficients.

PDE solution using FDM

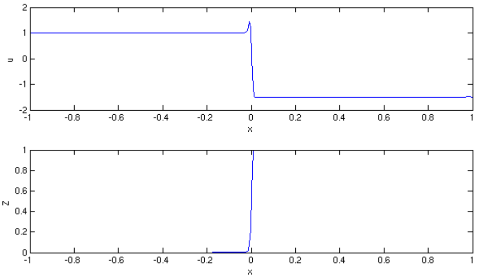


Figure: Solution for Reactive Shock Layer equation using finite differences

Optimization Procedure using FDM

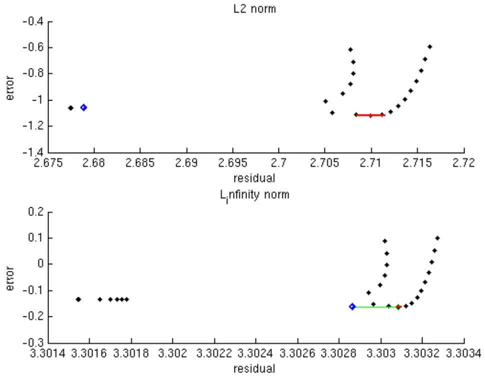


Figure: Optimization path for Reactive Shock Layer equation using finite differences

Solution using PPM

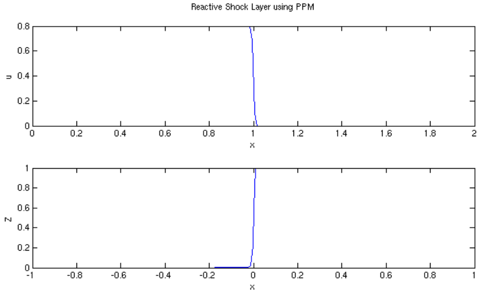


Figure: Solution for Reactive Shock Layer equation using PPM

Optimization Procedure using PPM

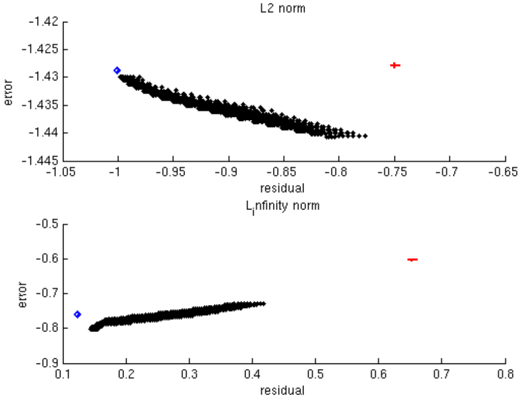


Figure: Optimization path for Reactive Shock Layer equation using PPM

Conclusions

- RE does not work on the fine grid G^*, but may work well on the coarse grid $G_{1,1}$ at time steps kdT, where dT is the coarse time step.
- One requires few SSOR smoothing of $\tilde{U}_{i,j}$ on G^* solutions to have OES performing better than the fine grid solution $\tilde{U}_{2,2}$.
- One can have OES better than RE for G^* and in the same time OES worst than RE as an approximation of the exact solution.
- the higher the order of the scheme, and/or the finer the discretization, the more iterates of SSOR we need.
- OES gives best results for under resolved solutions with low order scheme.
- smaller residual on G^* does not lead to smaller errors. The post-processing step with SSOR is then essential to recover this monotonic relationship between residual and errors.
- filtering the residual and/or the solution in space, might be beneficial for large time step.

General Principle (1)

- Let us assume that V_e exists and has been computed.
- Let U_j be one of the coarse grid approximations; We look for a global a posteriori estimate of the error

$$||\tilde{U}_j - p_h(u)||$$

- Recovery method:

$$\text{IF}||V_e - p_h(u)||_2 << ||\tilde{U}_j - p_h(u)||_2,$$
$$\text{THEN}||\tilde{U}_j - V_e||_2 \sim ||\tilde{U}_j - p_h(u)||_2$$

provides a good *lower* bound on the error in our numerical experiments with steady incompressible Navier Stokes (NS).

- But there is no guarantee that a smaller residual for V_e than for U_2 on the fine grid $M(h_\infty)$ leads to a smaller error.

General Principle (2)

- From a stability estimate with the discrete operator:

$$||V_e - U_\infty|| < \mu_\infty G(V_e), \text{where} \mu \geq ||(A_{h_\infty})^{-1}||.$$

- We conclude

$$||\tilde{U}_2 - U_\infty||_2 < \mu G(V_e) + ||V_e - \tilde{U}_2||_2.$$

- Uses extrapolation on μ_1, μ_2, μ_3 to get $\approx \mu_\infty$.
- L_2 norm: the estimate on μ uses a standard eigenvalue iterative procedure to get the smallest eigenvalue.
- L_1 norm: see N J.Higham papers.
- Additional Test: Verify that the upper bound on $||U_\infty - U_2||$ increases toward an asymptotic limit as $M(h_\infty)$ gets finer.
- Feasible test because the fine grid solution is never computed in OES.

General Principle (3)

- N non linear (discrete) operator from E to F.
- Assuming the problem $N(u) = s$ is well posed for $s \in B(S, d)$, and $N(U_h) \in B(S, d)$, for some discrete solution U_h.
- Defining ρ the residual and e the error, an upper bound of the error is given by

$$|| \, e \, ||_E \; \leq \; ||\rho||_F \; (||\nabla_s N^{-1}(S+\rho)||_E \; + \; \frac{K}{2} ||\rho||_F).$$

- Let $\{b_i^E, \; i = 1..N\}$, be a basis of E, and $\varepsilon \in \boldsymbol{R}$ such that $\varepsilon = o(1)$.
- Let $(V_i^{\mp})_{i=1..N}$, be the family of solutions of the following problems $N(U_h \mp \varepsilon V_i) = S + \rho \mp \varepsilon b_i$.
- We get from finite differences the approximation

$$||\nabla_S N^{-1}(S+\rho)|| \approx ||(\frac{1}{2}(V_j^+ - V_j^-))_{j=1..N}|| + O(\varepsilon^2).$$

Stability estimates

- $\hat{q}_E$ is least square approximation of the solution u in $\hat{E}$, acting as a filter on the solution
- q_E is a projection in E.

$$
\begin{tikzcd}
\underset{u}{E_h} \arrow[r, "C^{-1}"] \arrow[d, shift left, "\hat{q}_E"] & \underset{residual}{F_h} \arrow[d, shift left, "\hat{q}_F"] \\
\overset{\sum \alpha_i b_i}{\hat{E}_h} \arrow[u, shift left, "q_{\hat{E}}"] \arrow[r, "\hat{C}^{-1}"'] & \overset{\sum \beta_i c_i}{\hat{F}_h} \arrow[u, shift left, "q_{\hat{F}}"]
\end{tikzcd}
$$

The construction of q_E and $\hat{q}_E$, respectively q_F and $\hat{q}_F$ does not consider the nature of the approximation space of the code C since the implementation details are most of the time unavailable: the mappings involve only the discrete representations of the functions.

Algorithm :Idea

- Compact representation of unknown weight functions: m is much lower than the number of grid points on any coarse grid used.
- Estimate on the number of iterates to regularized $\tilde{U}_j, j = 1..p$
- Generalization to non-linear elliptic problems via a Newton like loop.
- Difficulties: A posteriori Error estimate depends then on the function used to linearized the operator.
- Generalization to L_1 and L_∞ with appropriate minimization procedure.

Algorithm(1)

- Let us denote $N[u] = 0$ the supposedly well posed PDE problem to be solved, and its unsteady companion problem, $\partial_t u = N[u]$.
- The algorithm is as follow:
- Step 1 *Call coarse Mesh* : We generate the (coarse) meshes G_1 and G_2. If h_i is the average space step for the grid G_i we should have $h_2 < h_1$ but this is not necessary.
- Step 2 *Call fine Mesh* : We generate a fine mesh G_∞ that is supposed to solve all the scales of the problem. G_∞ might be a structured mesh or not. We must have $h_\infty << h_1, h_2$.
- Step 3 *Call Solver* : We solve the problem on G_1 and G_2, possibly in parallel. The solutions are denoted respectively u_1 and u_2 on G_1 and G_2.

Algorithm(2)

- Step 4 *Call Projection* : We project these coarse solutions u_1 and u_2 onto G_∞. We denote these projections $\tilde{u}_1^\infty$ and $\tilde{u}_2^\infty$.
- Step 5 *Create sample* : We create sample solutions $u_\alpha^\infty = [\alpha \tilde{u}_1^\infty + (1-\alpha)\tilde{u}_2^\infty]$. We smooth out the spurious high frequency components of the build solution with few explicit time steps of $\partial_t u = N(u)$ starting from the initial condition: u_α^∞. The choice of the Optimum Design Space in which α is taken is one the main item of our research.
- Step 6 We compute the best α that minimizes the L_2 norm of the residual. We may use a surface response technique.

Adina Software

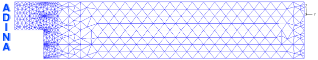


Figure: Coarse mesh

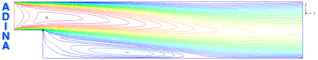


Figure: Contour of velocity magnitude on fine grid: Adina R&D

- In this simulation, the number of elements are respectively 10347 on the fine grid G^{∞}, 1260 on the coarse grid G_1, and 2630 on the coarse grid G_2. The steady solutions are obtained using a transient scheme for the incompressible Navier-Stokes equation.

Results

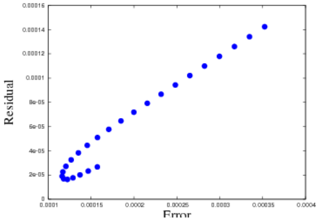


Figure: LSE: error and residual for Adina R&D in L_2 norm

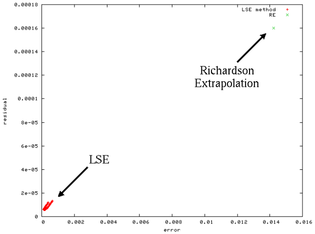


Figure: Performance of LSE and Richardson Extrapolation

Error bound for the back-step flow

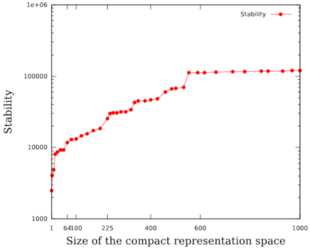


Figure: Evaluation of the stability constant

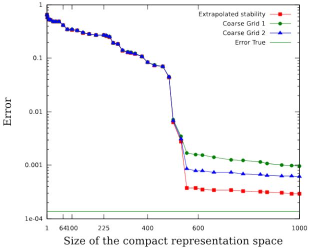


Figure: Evaluation of the error upper bound

Solution I

- The energy equation (9) that governs the model is:

$$\frac{\partial}{\partial x_i}\left(k_{ij}(T)\frac{\partial T}{\partial x_j}\right) + Q(T) = \rho c_p(T)\frac{\partial T}{\partial t} \quad \text{on } \Omega \times (0,t) \tag{9}$$

Solution II

- The boundary conditions

$$\bullet - \left(k_{ij}\frac{\partial T}{\partial x_j}\right) \cdot n = h_1(T)(T - T_\infty) + \sigma\varepsilon_1\left(T^4 - T_\infty^4\right) \text{ on } \Gamma_{N_1}$$

(radiation, convection)

$$\bullet - \left(k_{ij}\frac{\partial T}{\partial x_j}\right) \cdot n = h_2(T)(T - T_\infty) + \sigma\varepsilon_2\left(T^4 - T_\infty^4\right) \text{ on } \Gamma_{N_2}$$

(radiation, convection)

$$\bullet - \left(k_{ij}\frac{\partial T}{\partial x_j}\right) \cdot n = h_3(T)(T - T_\infty) \text{on } \Gamma_{N_3}$$

(convection)

•Symmetric boundary condition on Γ_{N_4}

Solution

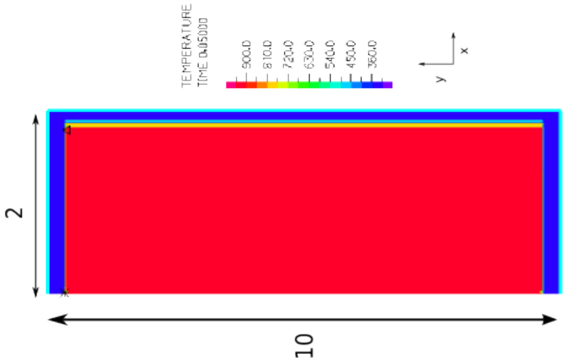


Figure: Steady state solution of the heat transfer problem.

Solution

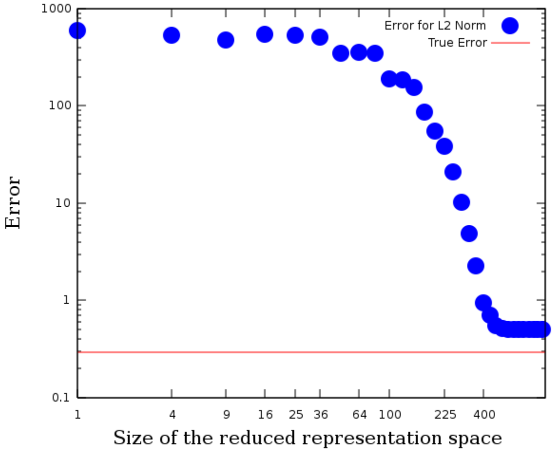


Figure: Evolution of the error versus the fine grid solution with the L_2 norm for the heat transfer problem

Computational cost

- In our procedure, there is a need to compute form a large number of solution in order ot perform the minimization.
- These computations have an embarrassing parallelism.
- On the other hand, given a code that is portable to different platform, there is a large amount of resources that are available.
- The question to be answered is can a distributed version of the verification procdeure be designed to take advantage of this two facts?

Architecture

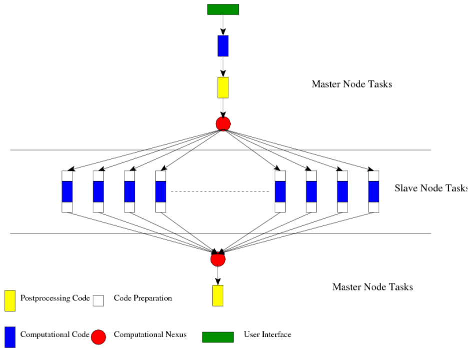


Figure: Software Architecture

Distributed performances

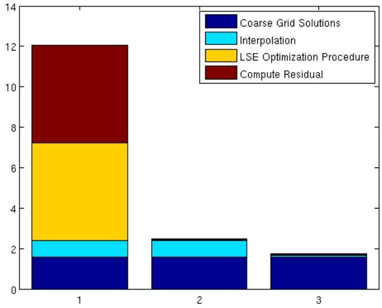
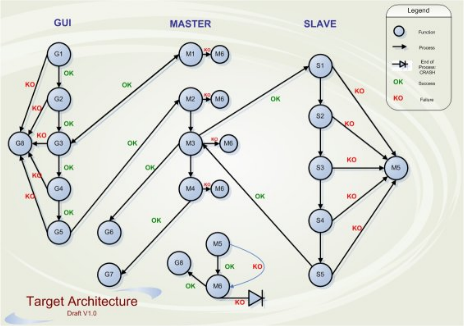


Figure: Time performances: OES versus computation of fine grid solution with a Pentium 4, 2.4GhZ, running a Linux OS.

Error control and secure data transfer



Conclusions

- A new extrapolation method for PDEs.
- A better tool for solution verification than RE when the convergence order is space dependent or far from the asymptotic rate of convergence.
- Solution Verification Method with Hands off Coding.

Perspectives

- Expand OES to other multiphysics case, ie Chimera method and IBM.
- Expand OES to compute stability estimates for unsteady problems.
- Integrate OES in applications as a plugin/tool.

Publications I

- Conference
  1. Aitken like acceleration of the Schwarz algorithm for overset methods: application to Incompressible Navier Stokes flow - 7th Symposium on Overset Composite Grid and Solution Technology - Huntington Beach, California, USA , October 5-7, 2004
  2. Solution Verification on a Grid - C. Picard, M. Garbey and V. Subramanian - Workshop on Parallel Computing - April 9, 2005, Houston, Tx
  3. Heterogeneous Domain Decomposition for multi-scale problems - M. Garbey, C. Picard, F and Pacull - Multiscale Simulation of Coupled Physical Problems - Santorini in Greece, May 25-28, 2005.
  4. A Numerical Approach for Multi-Scale Biomedical Problems - M. Garbey, C. Picard and R. Tran-Son-Tay - 2005 BMES Annual Fall Meeting, Sep 28-Oct 1, Baltimore, Md

Publications II

5. Solution Verification-Thanks to Distributed Computing - C. Picard, M. Garbey and V. Subramanian - Computational Science 2006 Workshop - March 3, 2006, Houston, Tx
6. Extrapolation method and optimum design of solutions - C. Picard and M. Garbey - 7th World Congress on Computational Mechanics - Los Angeles, Ca - July 16 - 22, 2006
7. Solution Verification in CFD and Heat Transfer - C. Picard and M. Garbey - Third Annual Workshop on Interdisciplinary Computational Science - Houston, Tx - March 22-24, 2007

- Proceedings

1. B.O. Dia, M.Garbey ,C. Picard and R. Tran Son Tay. Heterogeneous Domain-Decomposition for Multi-Scale Problems. 43rd AIAA Aerospace Sciences Meeting and Exhibit. 10 - 13 Jan 2005 - Reno, Nevada. AIAA-2005-1092.
2. M.Garbey and C. Picard. A least square extrapolation method for heat transfer. 16th International Conference on Domain Decomposition Methods. New York City, January 12-15, 2005.

Publications III

3. Mapping LSE Method on a Grid: Software Architecture and Performance Gain - C.Picard, M.Garbey and V.Subramanian - May 24 - 27, 2005 - University of Maryland, College Park Campus
4. M.Garbey and C.Picard, A Least Square Extrapolation Method for the a priori Error Estimate of CFD and Heat transfer Problem, C.Soize, G.I.Schueller Editors, pp871-876, Structural Dynamic Eurodyn 2005
5. A Multilevel Method for Solution Verification - M. Garbey and C. Picard - 17th International Conference on Domain Decomposition Methods - St. Wolfgang-Strobl, Austria, July 3-7 2006
6. A General Solution Verification Method for Complex Heat and Flow problem with Hands off Coding - C. Picard and M. Garbey - The 5th International Conference on Computational Heat and Mass Transfer (ICCHMT) - Canmore, Alberta, Canada - June 18-22, 2007

Publications IV

7. Parallel implementation for solution verification of CFD code - C. Picard and M. Garbey - 16th International Conference on Software Engineering and Data Engineering - Las Vegas, Nv - July 9-11, 2007

- Journal
  1. M.Garbey and C.Picard, Toward a General Solution Verification Method for Complex PDE problem with Hands of Coding, Computer and Fluids - Submitted
  2. C.Picard and M.Garbey, Optimized extrapolation method for parabolic equations. In preparation
  3. C.Picard and M.Garbey, Distributed computation of optimized extrapolation. In preparation

Activities

1. Investigation on numerical method for heterogeneous domain decomposition.
2. Investigation on numerical method for fluid-structure interaction
3. From Fall 2006 to Fall 2007 : Instructor for COSC 3661 and COSC 3662 (Numerical Analysis)
4. Developement of the Intelligent Data and Visualization Desk : Application for a patent 2483-00501

- **Thank you**