



Quelques exemples d'écoulements confinés en milieu poreux et problèmes associés

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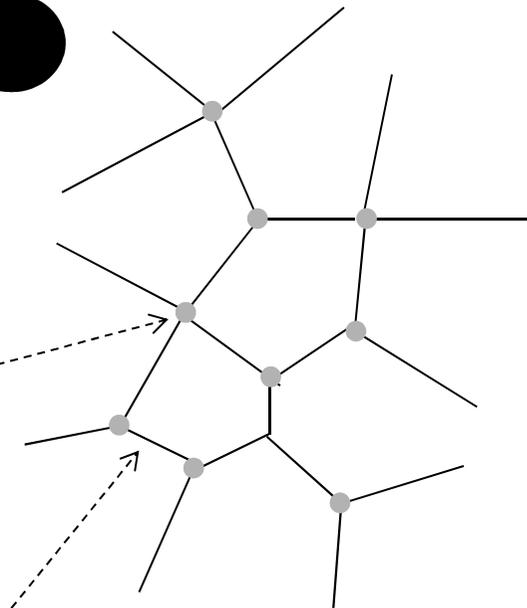
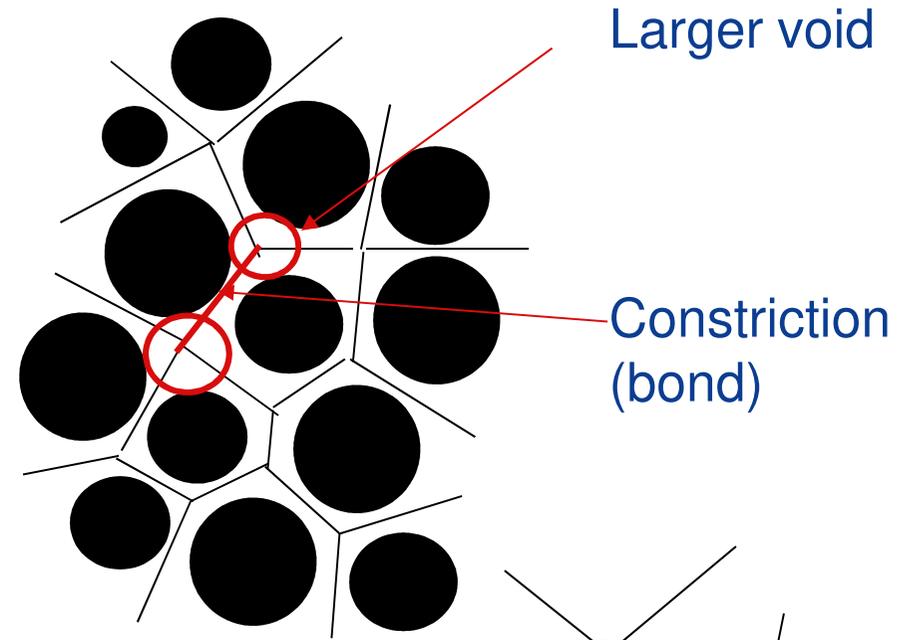
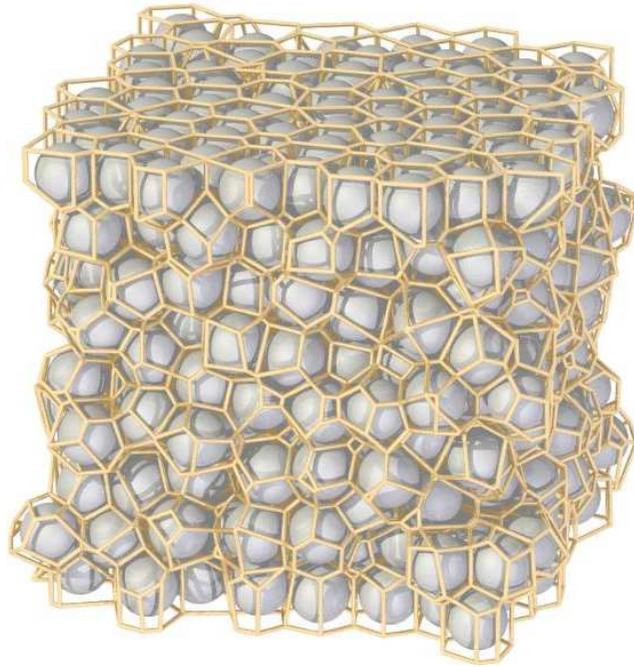
Sommaire

- Exposant de percolation et effet de raréfaction ?
- fort désordre / fort gradient
- Loi d'imbibition en nanofente
- Evaporation : diffusion Knudsen ? + Théorie HKS micro et macro.
- Couplage écoulements –déformations (argilite).

Sommaire

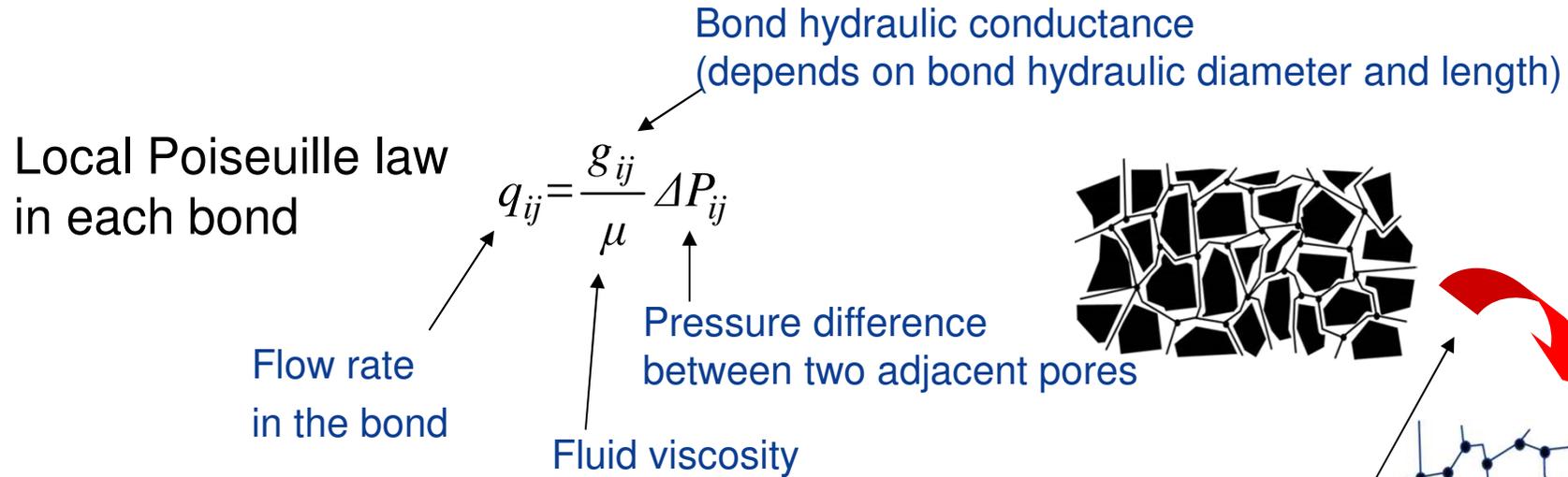
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Pore network



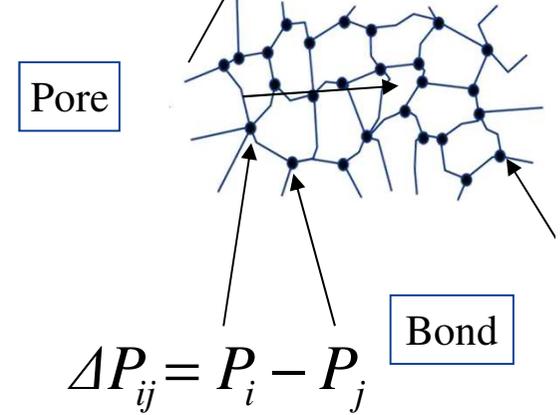
Porous medium \approx network of sites (pores) and bonds (throats, constrictions)

Example: computation of pressure and velocity fields in a network



Mass conservation at each node (pore) : $\sum_j q_{ij} = 0$

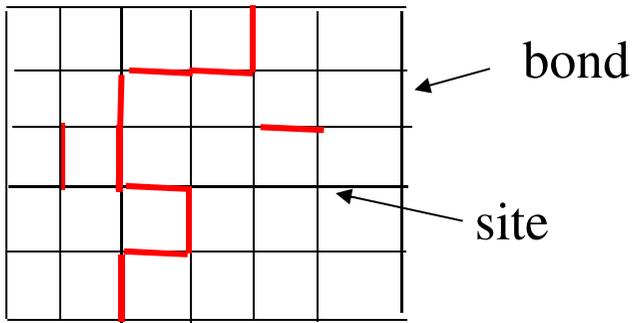
(numerical solution of linear system gives pressure
in each node (pore) of network)



➔ Random resistance network

Kirshhoff's circuit laws

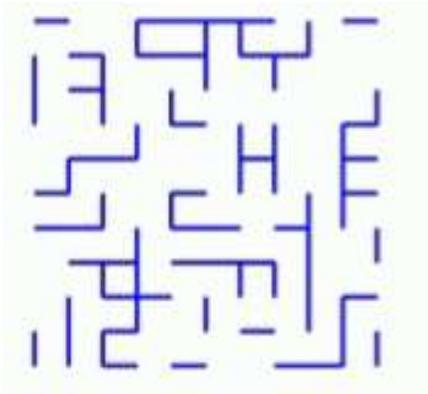
Percolation:



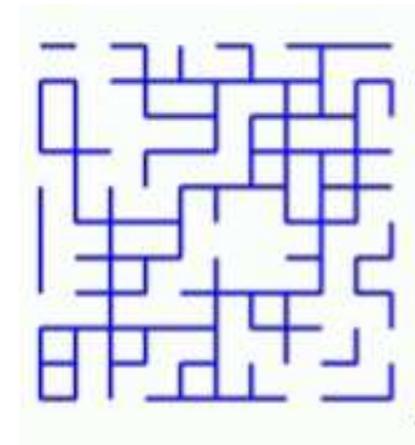
- very large network
- fraction p of conducting (active) bonds distributed randomly

*Percolation threshold p_c

$p < p_c$ finite clusters,
no long distance
connexion

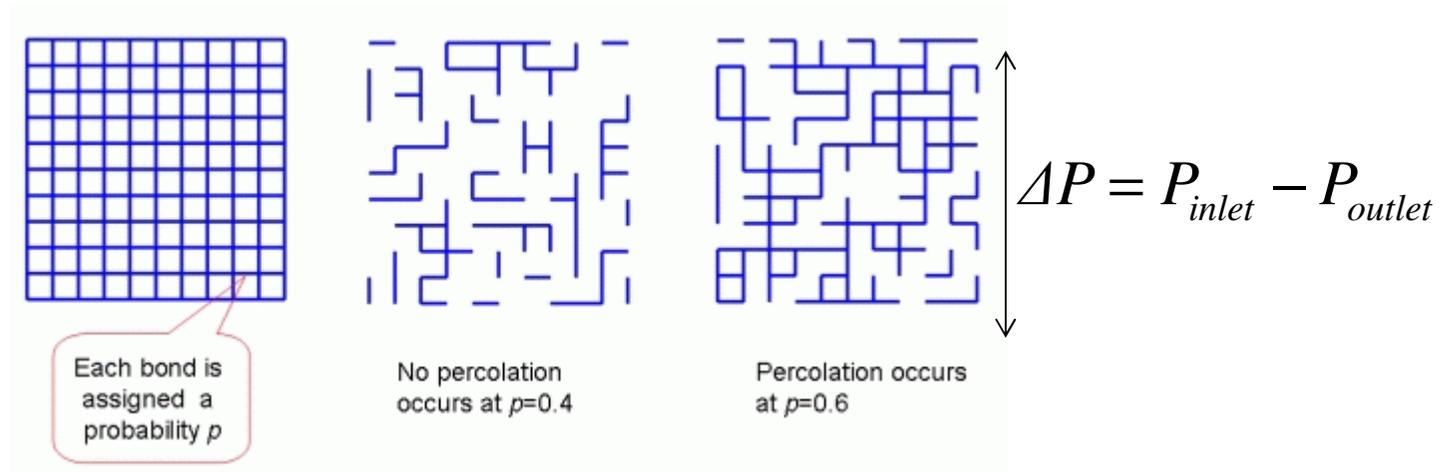


$p \geq p_c$
infinite cluster



Lattice	p_c (bond)
Square	0.5
triangular	0.34729
Simple cubic	0.2488

Percolation:



Leak (m^3/s)

$$Q = \frac{G}{\mu} \Delta P$$

G is the network global conductance

$$G \propto (p - p_c)^t$$

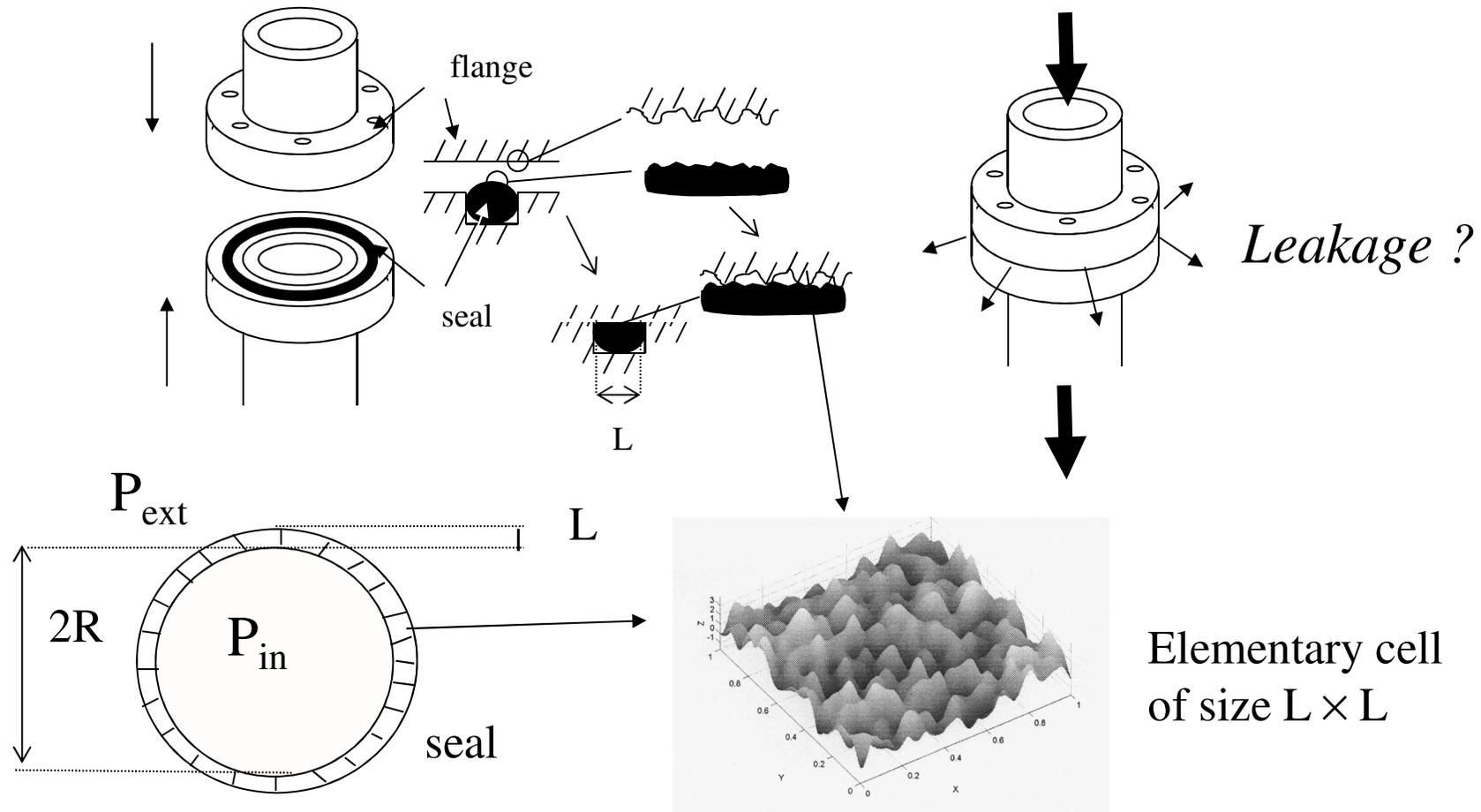
Power law behavior near p_c

(« universal » exponent $t = 1.3$)

Pore Network Model (PNM) for leak computation
between rough surfaces:

- A « perfect » PNM

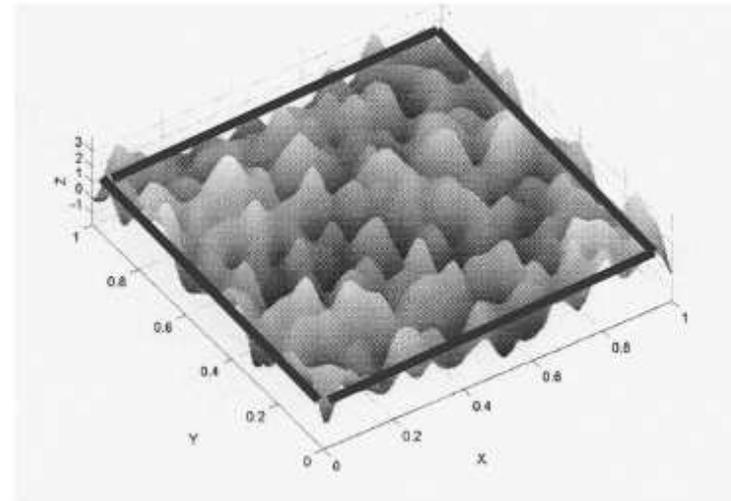
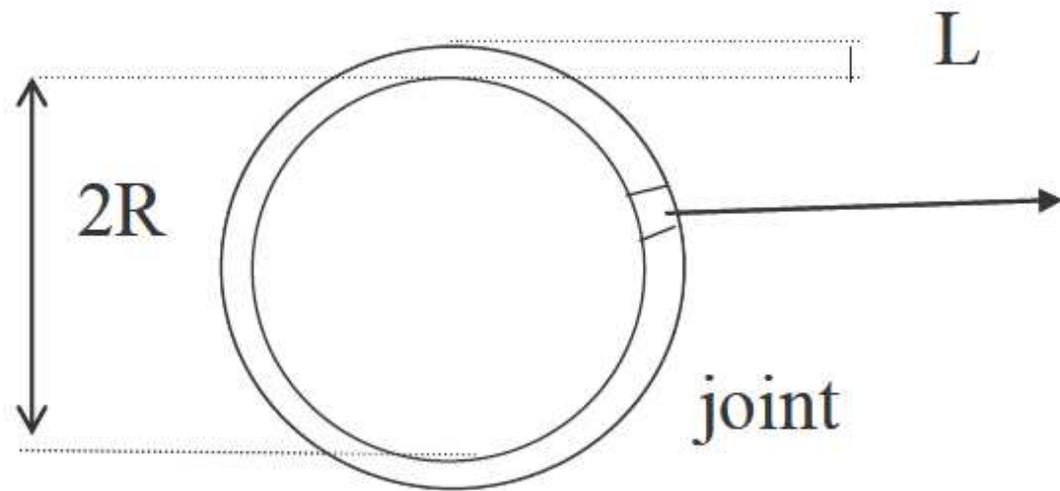
Hydraulic Conductance of a Metallic Static Seal (short range correlated isotropic roughness)



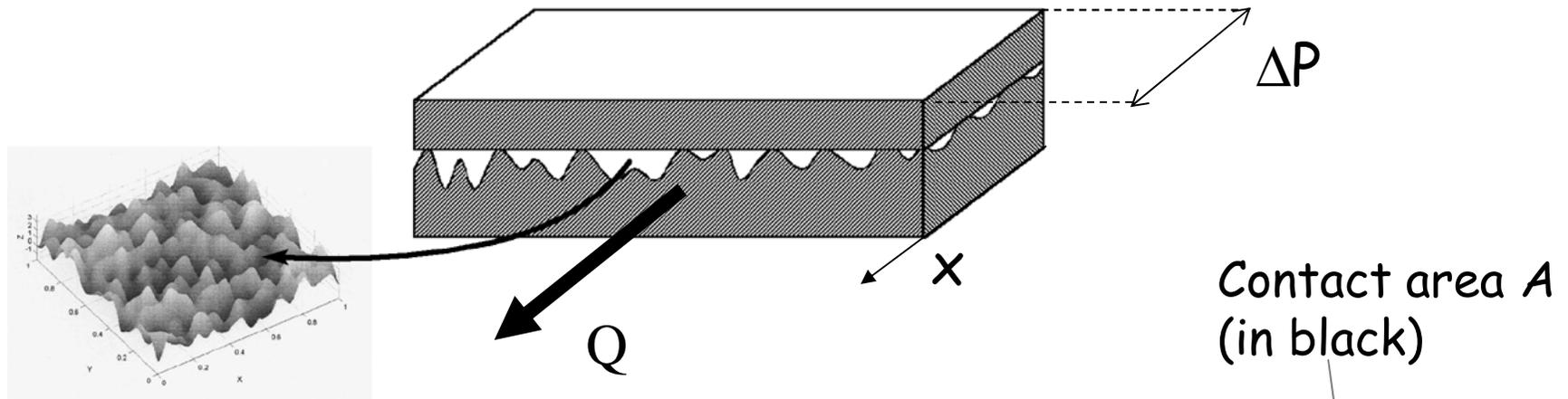
$$\Delta P = P_{in} - P_{ext}$$

Two scales : seal scale and elementary cell scale

Global conductance at the scale of a unit cell



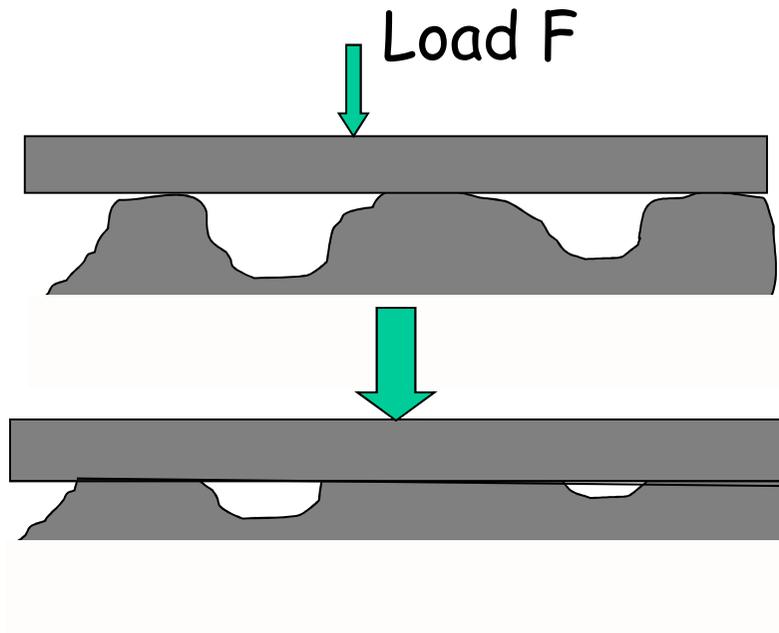
Static Seal conductance (isotropic random roughness)



$$Q = \frac{G}{\mu} \Delta P$$

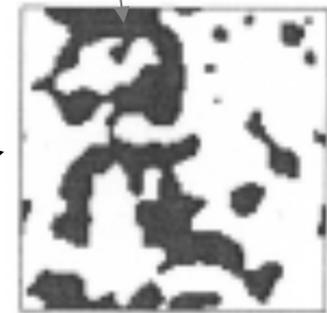
$$G = f(A) ?$$

Global
conductance

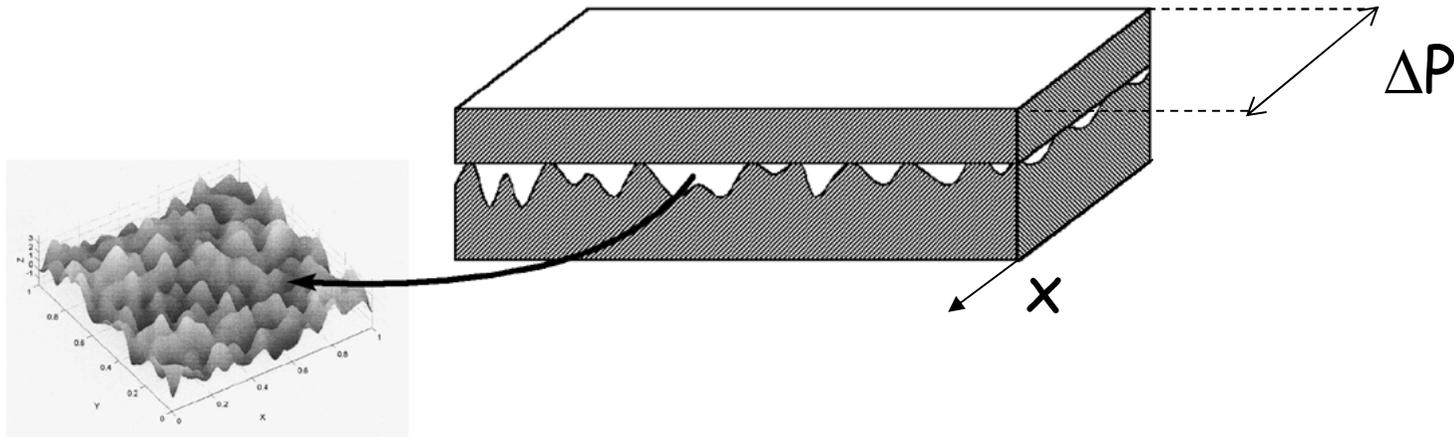


Contact
area

Contact area A
(in black)



Static Seal conductance (isotropic random roughness)



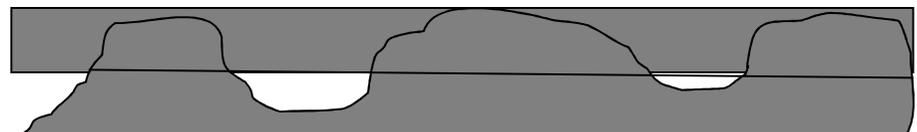
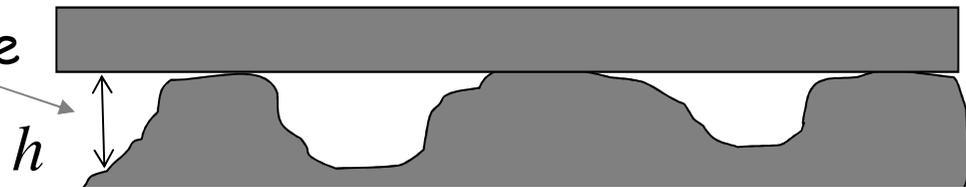
« *Microscopic* » model

$$q = -\frac{h^3}{12\mu} \nabla p$$

$$\nabla \cdot q = 0.$$

Lubrication equations

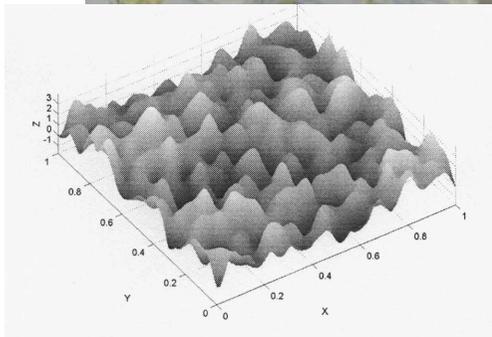
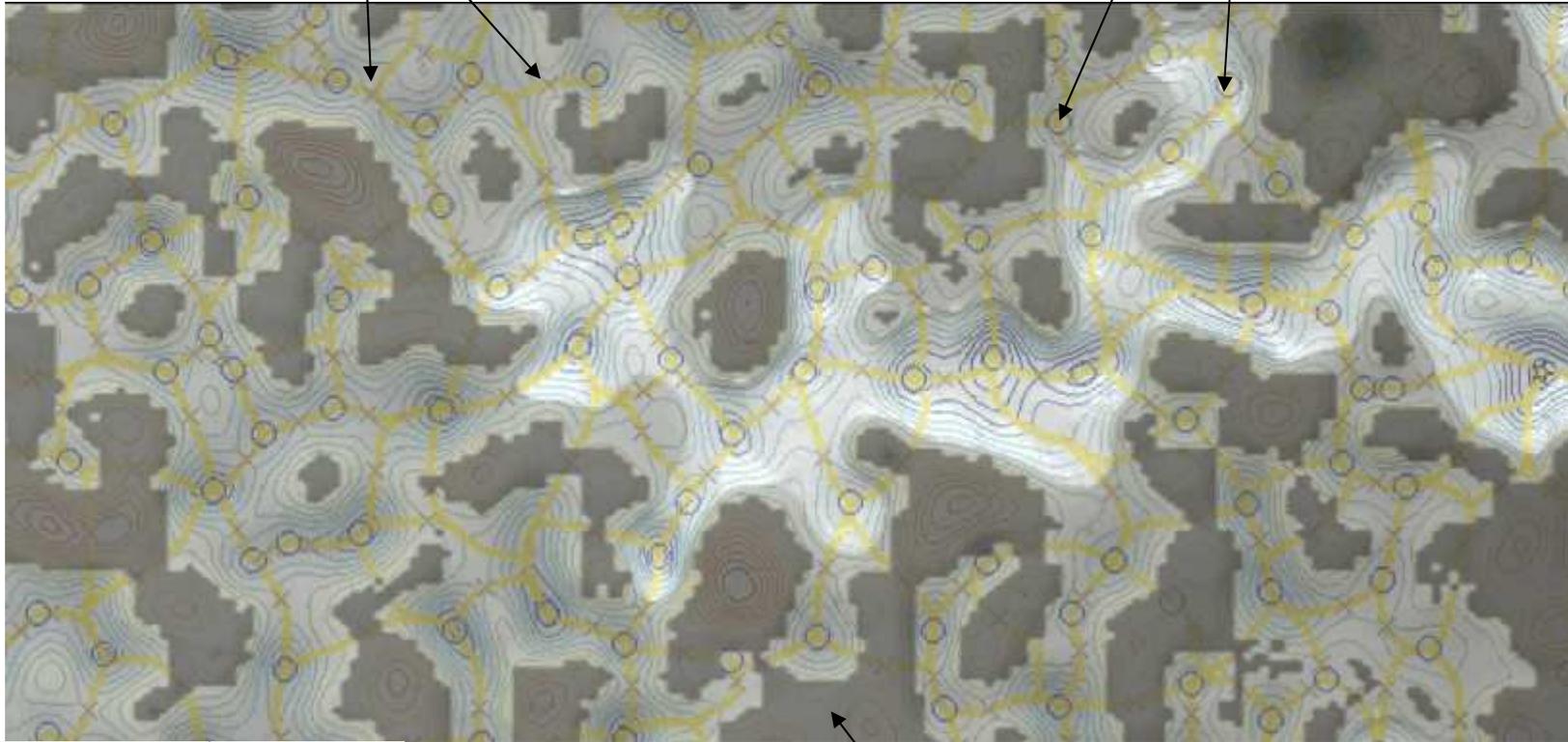
Local aperture



Network model

Saddle points (constrictions)

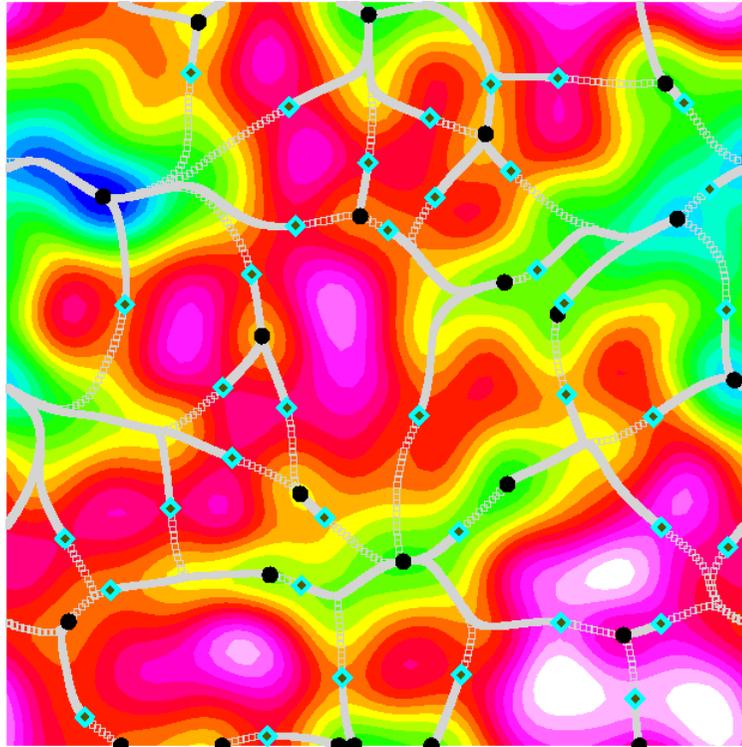
Aperture maxima



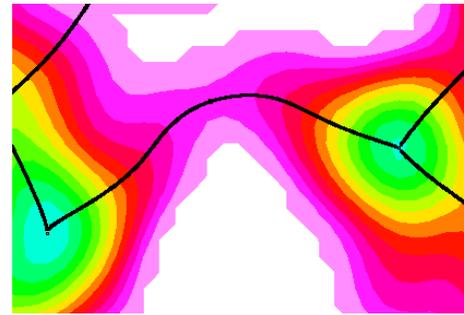
Short range correlated gaussian surface

Contact area
(dark grey)

Network model



Aperture field as a bond/site network

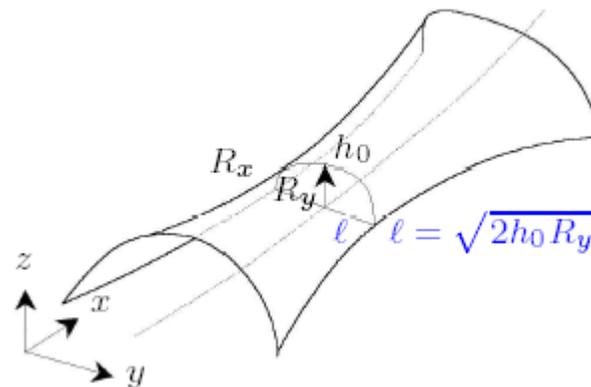


Bond detail

=
1 saddle point + 2 maxima

Hydraulic conductance of a bond

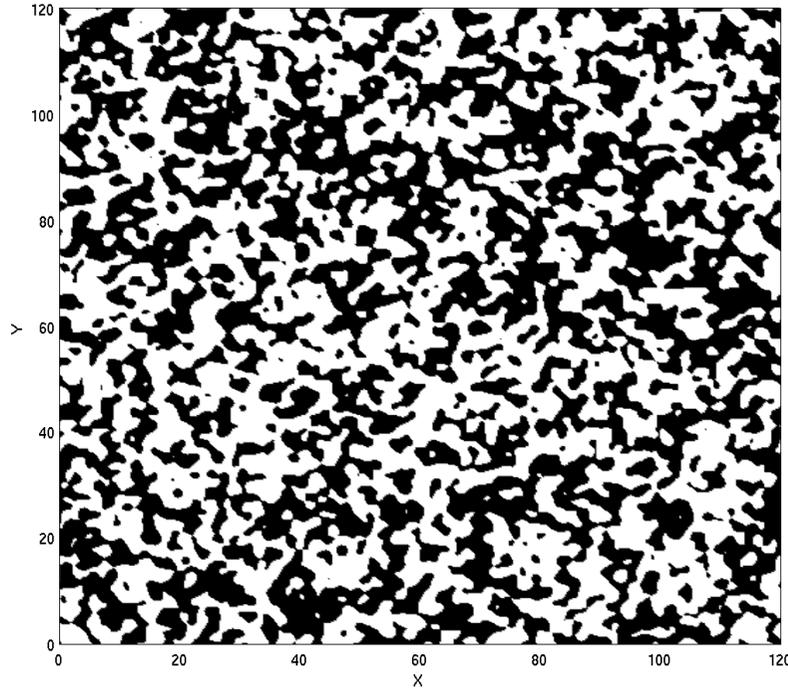
$$q = \frac{g}{\mu} \delta p \quad g = \frac{1}{14} h_0^3 \sqrt{-\frac{h_{xx}}{h_{yy}}}$$



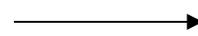
$$\varepsilon = \sqrt{\frac{h_0}{R_x}}$$

Small parameter

« Perfect » network approach



Continuum aperture field

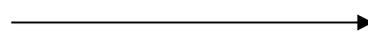


Aperture field as a
bond/site network

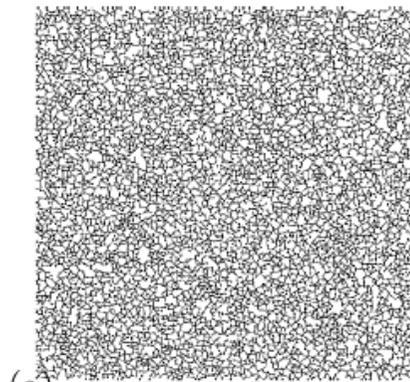
(Asymptotic representation)

$$\varepsilon = \sqrt{\frac{h_0}{R_x}} \longrightarrow 0$$

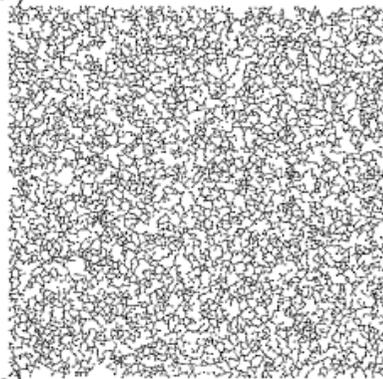
2D lubrication equation



Random resistor network
problem



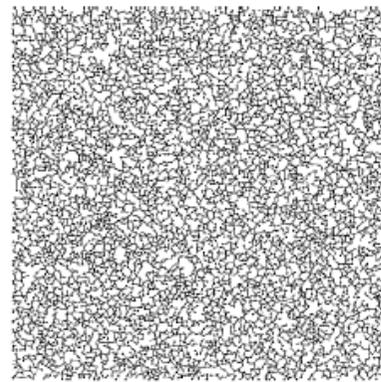
(a)



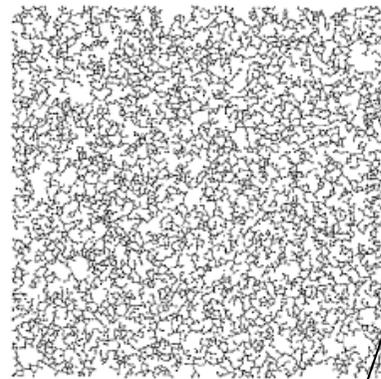
(c)



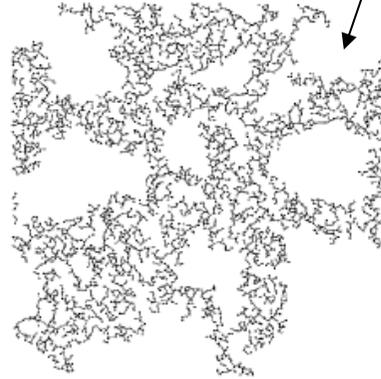
(e)



(b)

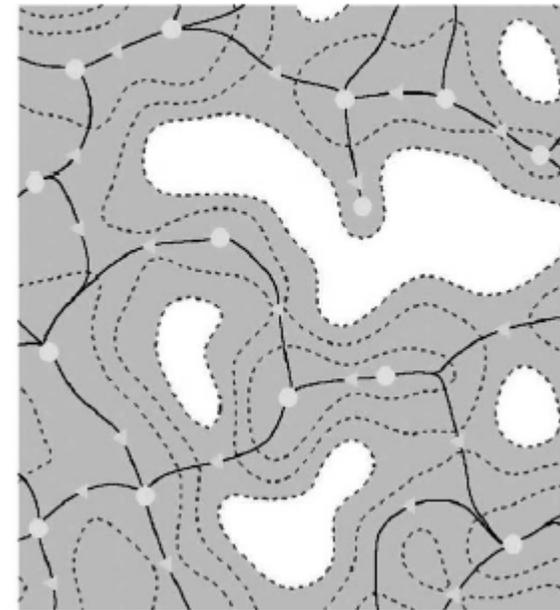


(d)



(f)

Contact area (in white)
increases up to
 A_c (percolation threshold)
where $G = 0$.



Hydraulic conductance behavior near percolation threshold

$$Q = \frac{G}{\mu} \Delta P$$

Percolation theory: $G \propto (A_c - A)^t$

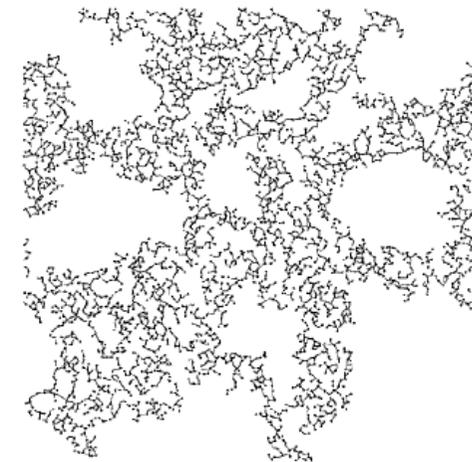
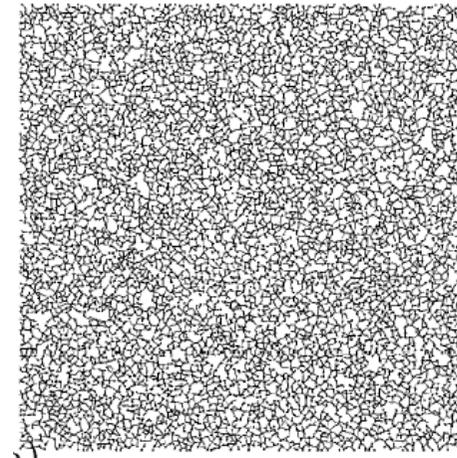
A contact area

A_c contact area at percolation threshold

PN simulations \Rightarrow

$$t = 2.8 \pm 0.05.$$

Non universal behavior
(« universal » exponent $t = 1.3$)



Why a non universal behavior ?

4.2.2.2.3 Distribution de probabilité des conductances locales

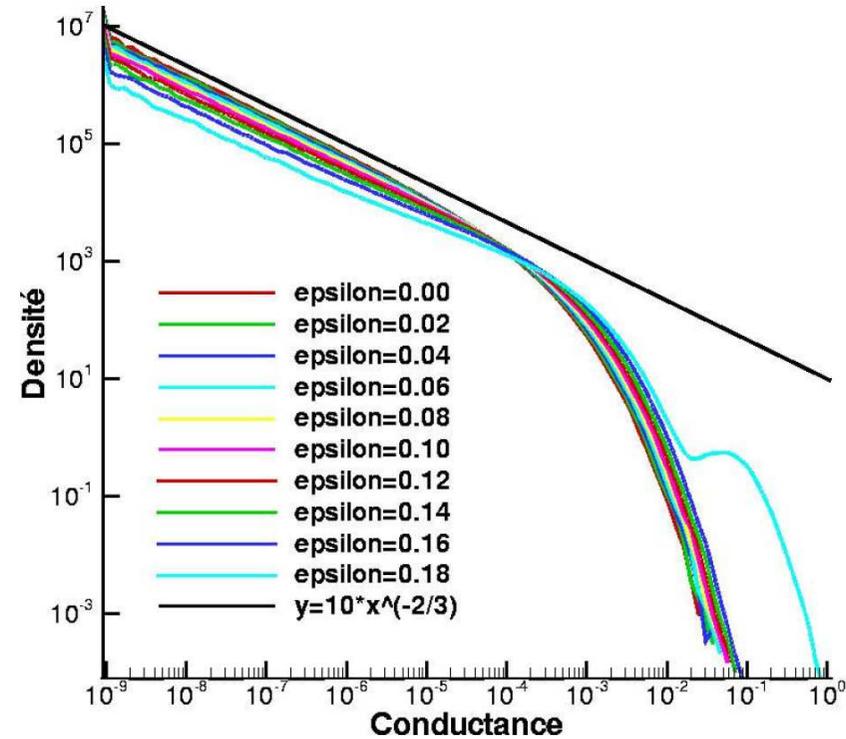
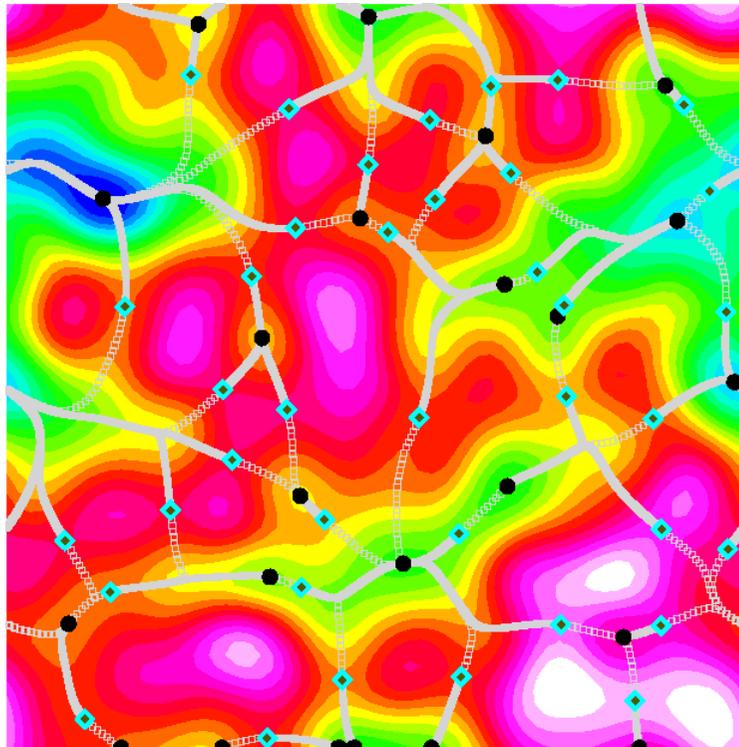
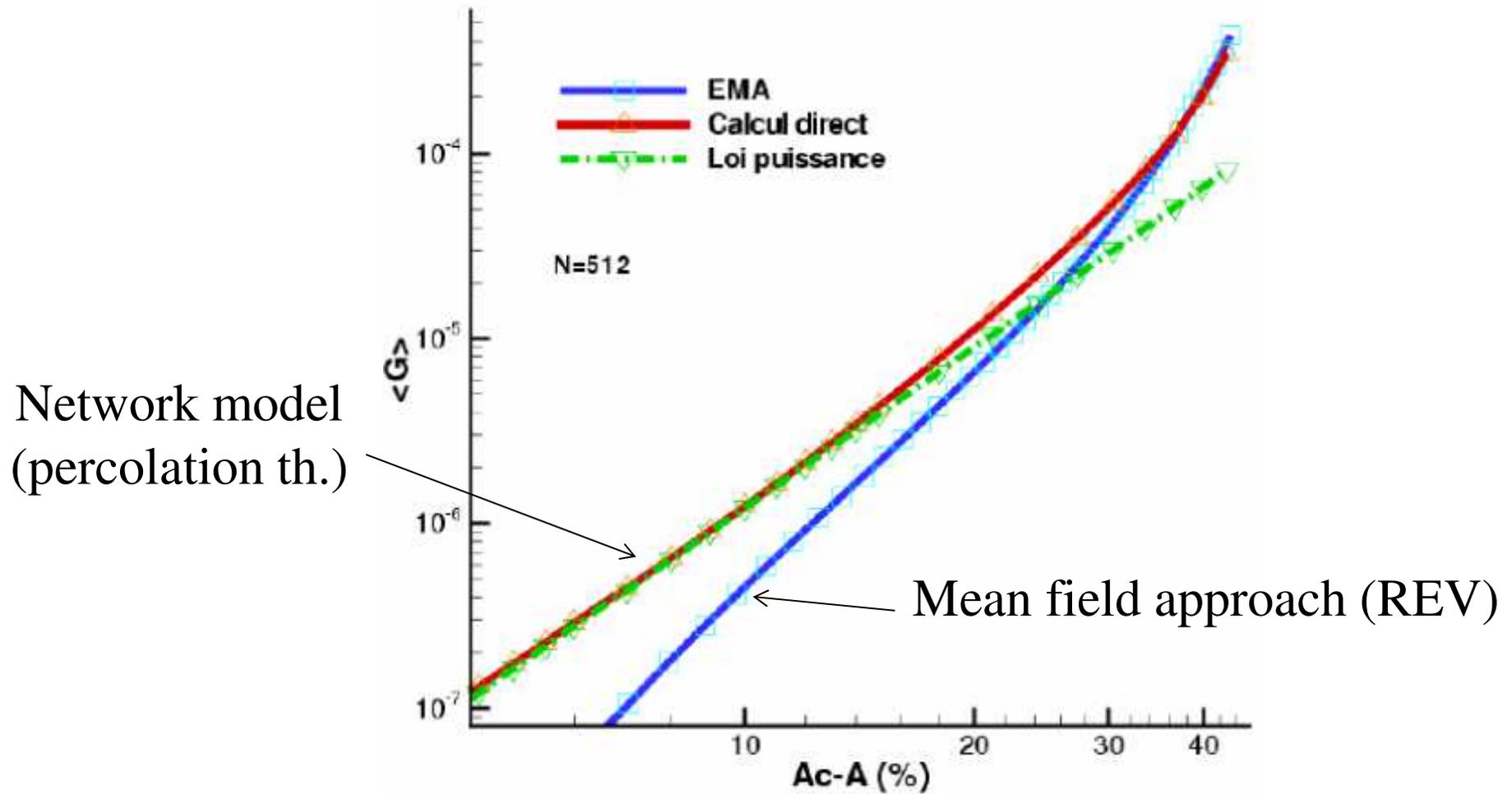


Figure 4.6. Distribution des conductances locales pour différentes tailles de pavé.
 $\varepsilon = (L - L_c) / L_{tot}$ où L est le nombre de liens actifs dans le réseau, L_c le nombre de lien actif au seuil de percolation, L_{tot} le nombre total de liens dans le réseau avant contact.

p.d.f. local conductance follows a power law over a large range of values near percolation threshold

Unit cell average global conductance

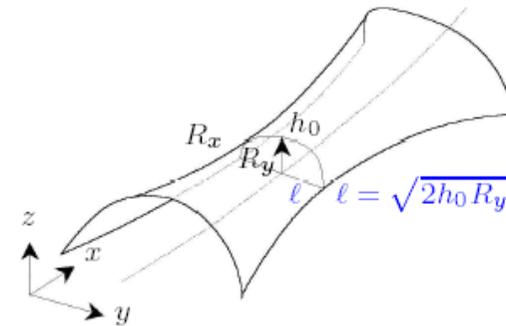


Questions ouvertes:

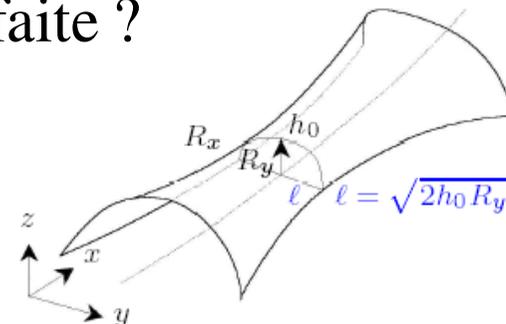
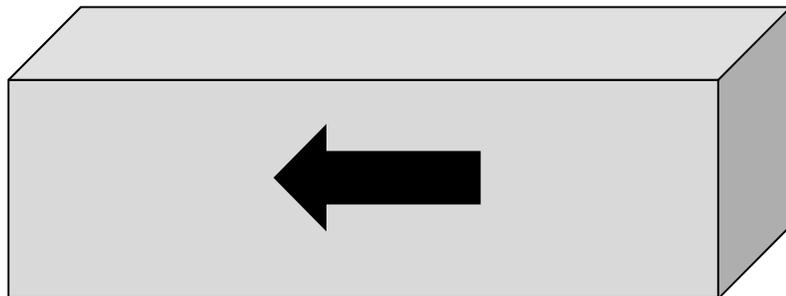
Q1: Impact de la raréfaction sur l'exposant ? Toujours 2.8 ?

Q2: La loi locale dépend –elle toujours uniquement de la géométrie du col en présence d'effet de raréfaction ?

$$q = \frac{g}{\mu} \delta p \quad g = \frac{1}{14} h_0^3 \sqrt{-\frac{h_{xx}}{h_{yy}}}$$



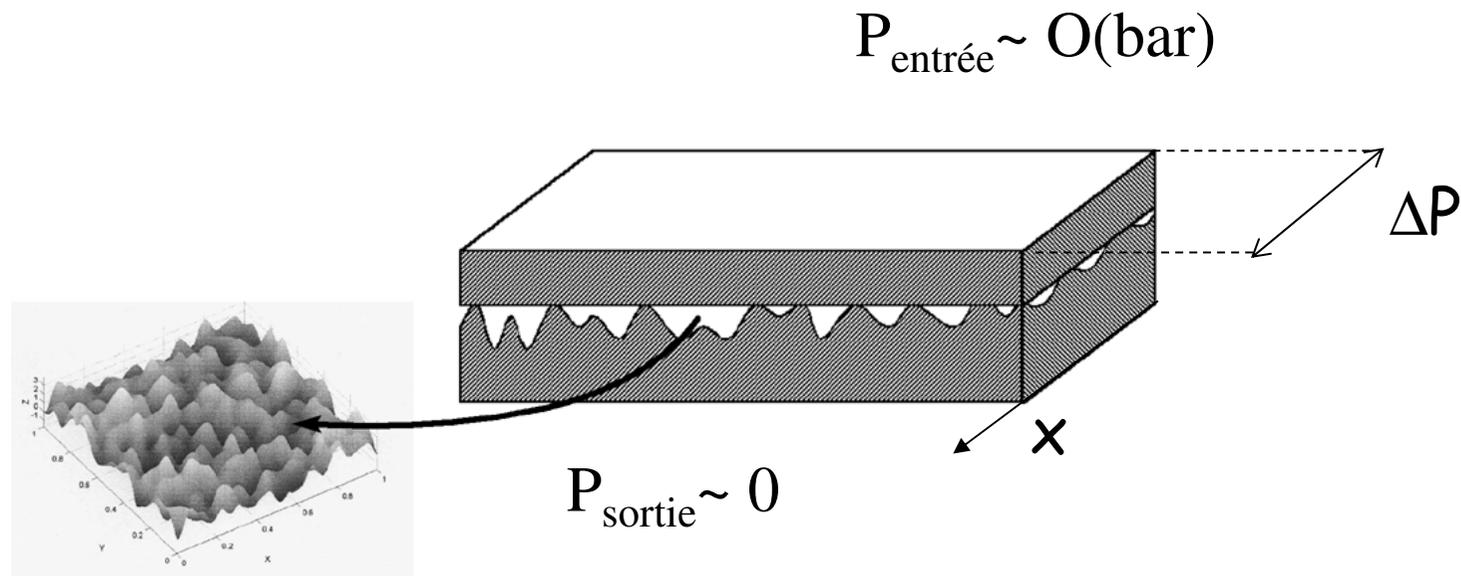
Q3: Loi locale d'écoulement en raréfié lorsque l'invariance par translation n'est plus satisfaite ?



Sommaire

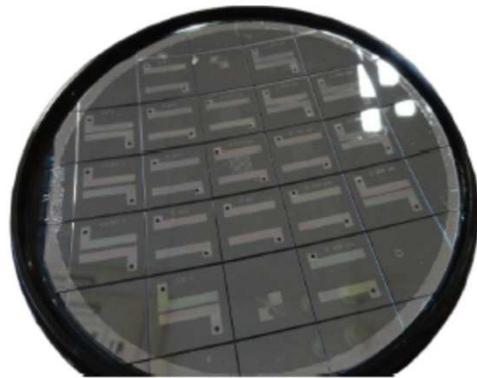
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fort désordre / fort gradient



Régime visqueux en entrée Régime à grand Kn en sortie

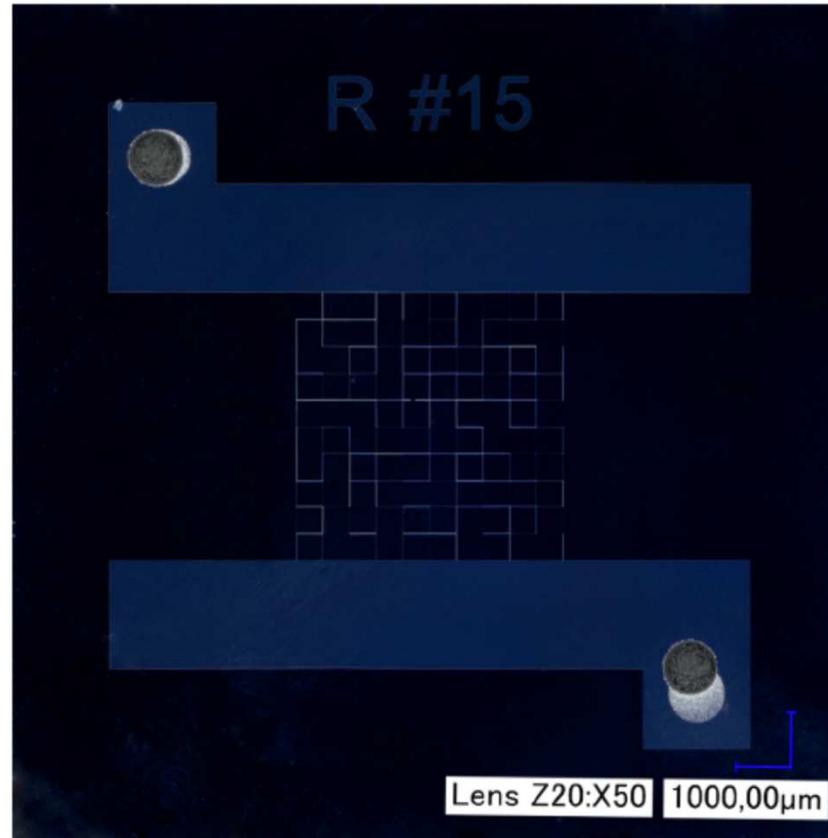
Puces nanofluidiques



(a) *Wafer* complet.



(b) Échelle avec une pièce de 1 €.

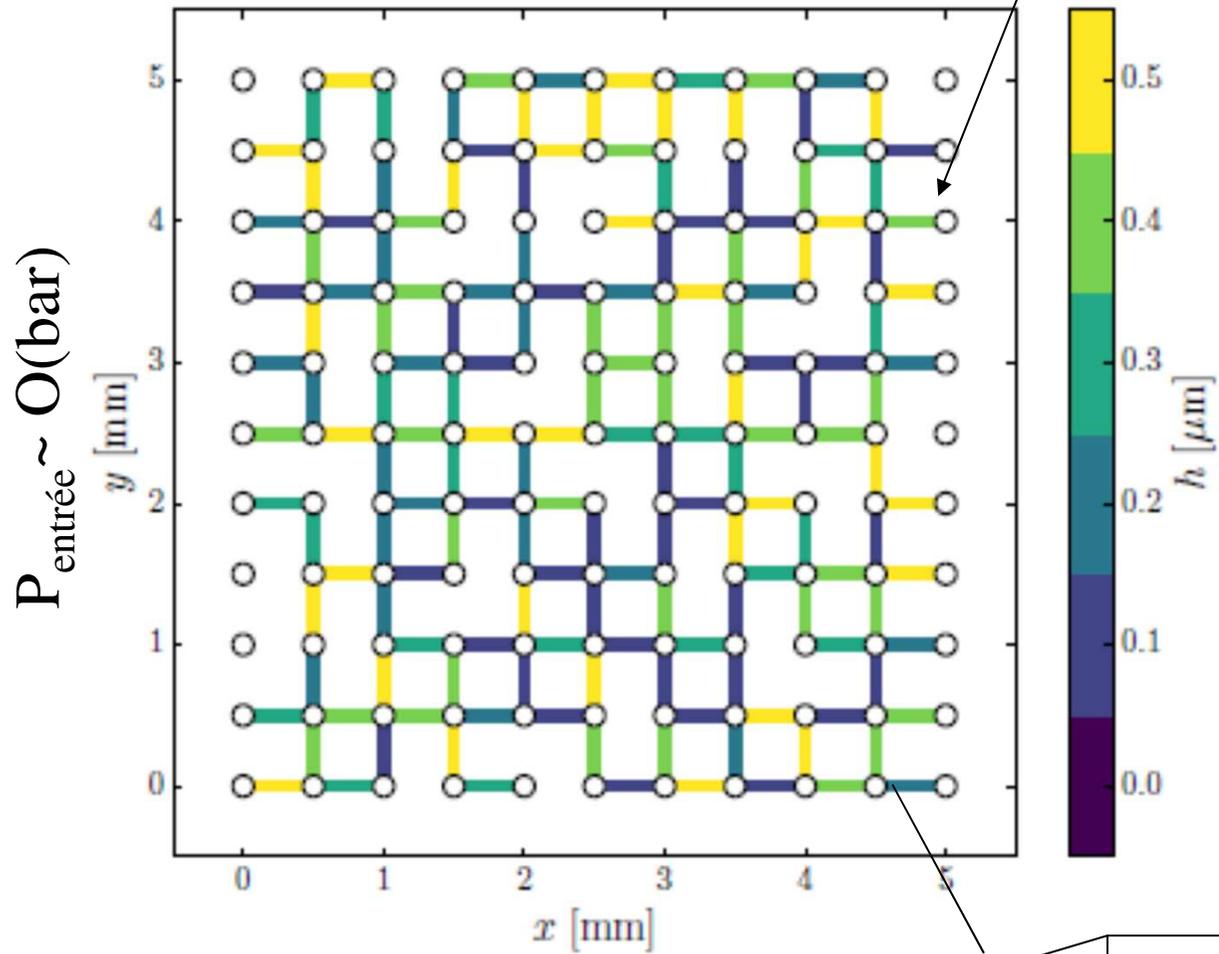


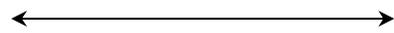
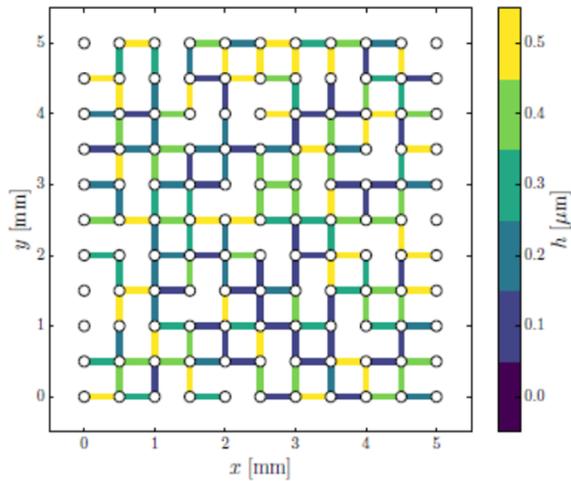
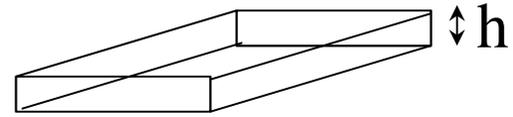
(c) Puce réseau hétérogène « R#15 ».

FIGURE 2.10 – Illustrations de quelques puces nanofluidiques fabriquées.

Réseau interconnecté de fentes

$P_{\text{sortie}} \sim 0$

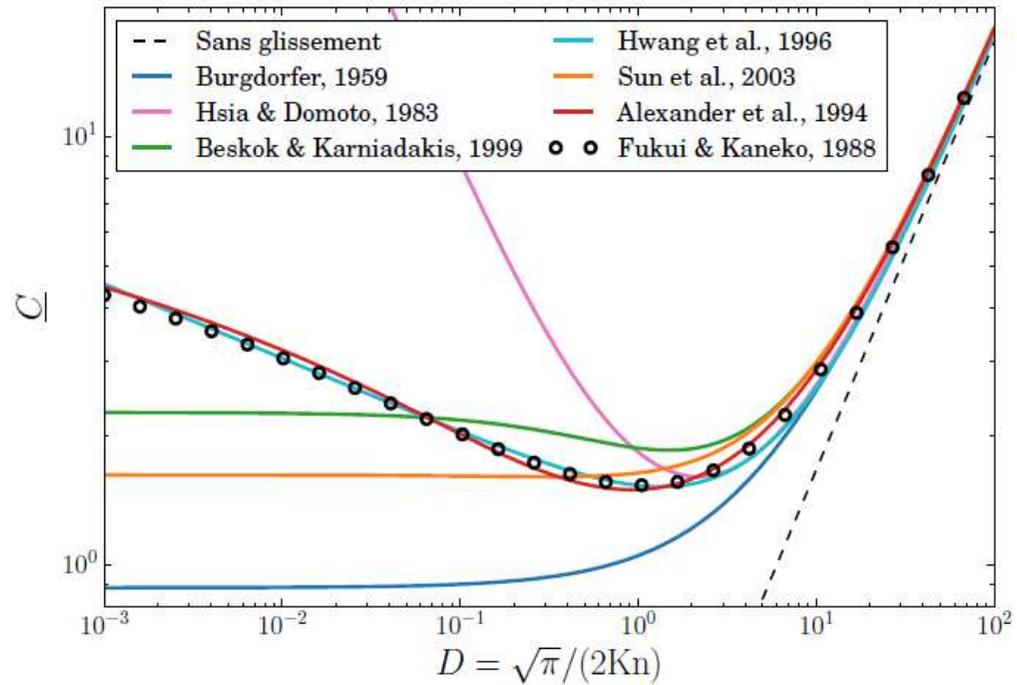




$$\Delta P = P_{\text{entrée}} - P_{\text{sortie}}$$

$$O(\text{bar}) \quad \sim 0$$

Q à l'échelle globale ?



Loi locale (débit / ΔP pour une fente)

$$Q_p = -\rho \frac{h^3}{12\mu} Q^* \frac{dp}{dx} \quad \underline{C} = \frac{\sqrt{\pi}}{12Kn} Q^* = \frac{Q^* D}{6}$$

Voir présentation T.Zaouter

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Imbibition de nanofentes

Nanofentes

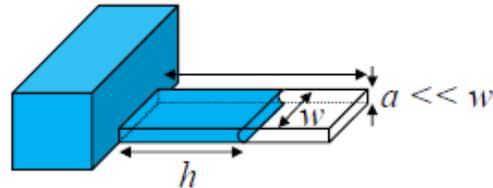
Invasion capillaire

Dimensions des nanocanaux réalisés :

a : 20 à 390 nm

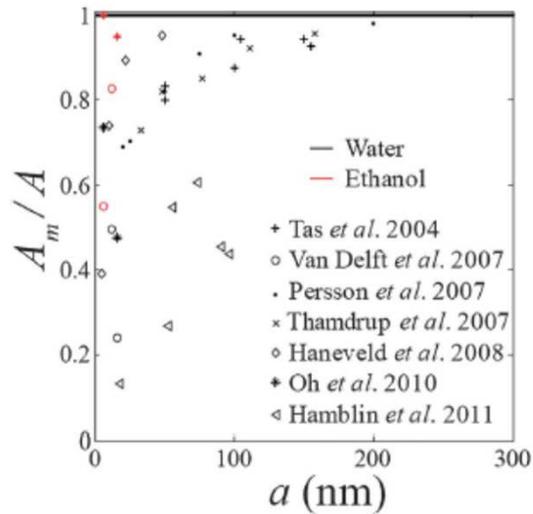
w : 3 à 10 μm

L : 200 μm à 5 mm



$$h = \sqrt{At}, A = \frac{a \gamma \cos \theta}{3\mu}$$

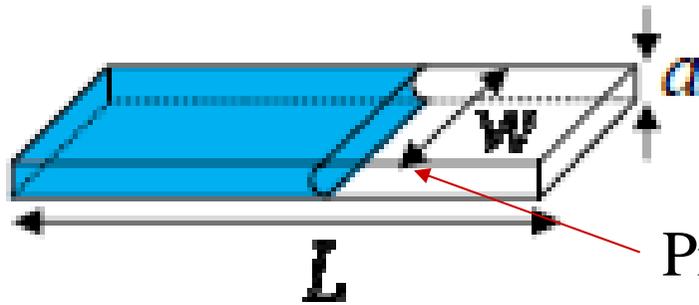
Washburn, 1921



Imbibition plus lente
que loi de Washburn ?

Fig. 1 Capillary filling slowdown in nanoslits reported by several authors.^{35,37-40,45,56} Results are expressed as the ratio A_m/A of the measured coefficient A_m to the predicted one A in Washburn law $h^2(t) = At$.

Imbibition de nanofentes



$$\frac{\partial p_g}{\partial t} = \frac{\partial}{\partial x} \left((p_g + p_a) \frac{a_g^2}{12\mu_g} \frac{\partial p_g}{\partial x} \right)$$

Pressurisation du gaz ?

$$t_p = L^2 / D_g$$

$$D_g \approx p_a a^2 / 12\mu_g$$

Effets de films

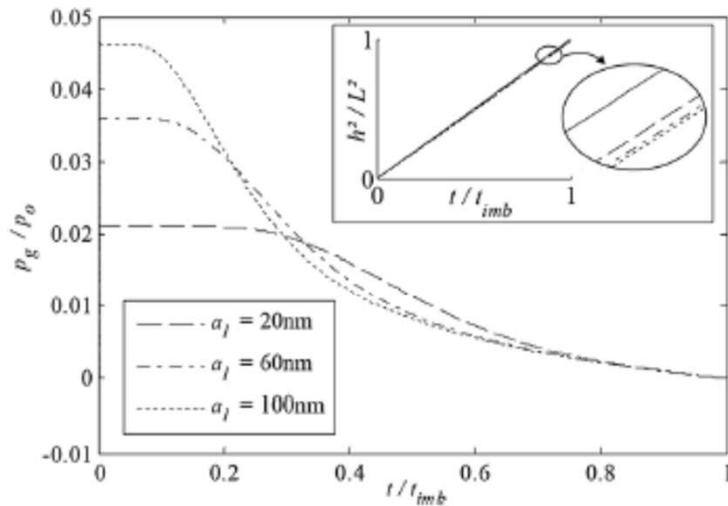
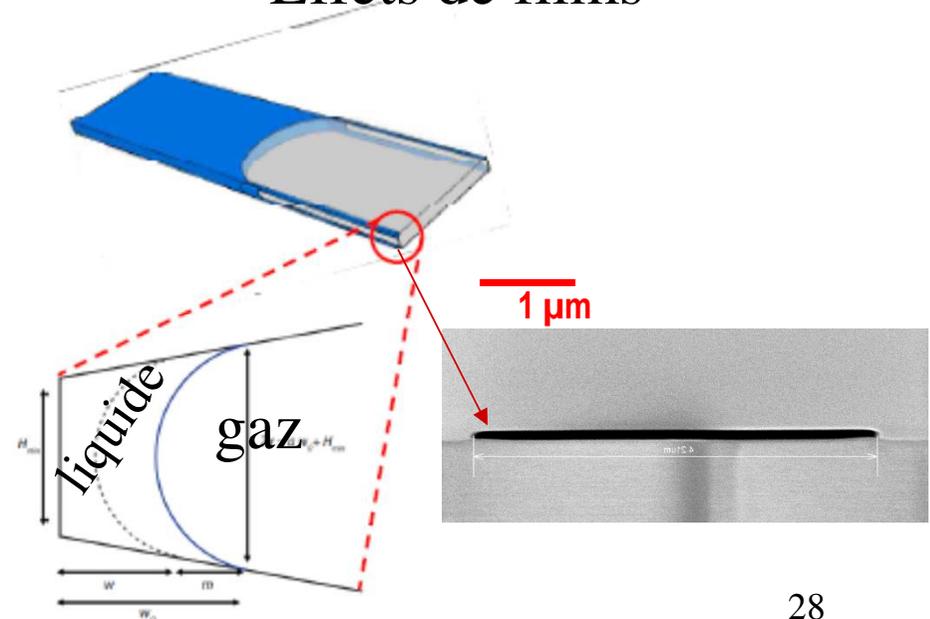


Fig. 11 Evolution of gas pressure at the front during the imbibition



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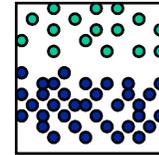
EVAPORATION

Résistance interfaciale

Evaporation = vaporisation + phénomènes de transport

Equilibre thermo.= équilibre dynamique entre molécules se condensant et s'évaporant

Théorie cinétique:



Evaporation = rupture de cet équilibre dynamique (flux de masse net à l'interface)

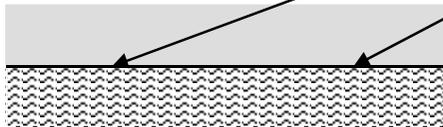
•L'interface n'est pas à l'équilibre thermodynamique lorsqu'il y a changement de phase

•Il existe une valeur max du flux d'évaporation

$$n_v = \alpha \left(\frac{M}{2\pi RT} \right)^{1/2} (P_{ve} - P_{vi})$$

$$\alpha = \left[\frac{2\sigma}{2 - \sigma} \right]$$

σ est le coefficient d'accommodation



EVAPORATION

Résistance interfaciale

$$n_v = \alpha \left(\frac{M}{2\pi RT} \right)^{1/2} (P_{ve} - P_{vi})$$

$$n_v = \frac{(P_{ve} - P_{vi})}{R_i} = \frac{DM_v}{RT} \frac{(P_{ve} - P_{vi})}{\delta}$$

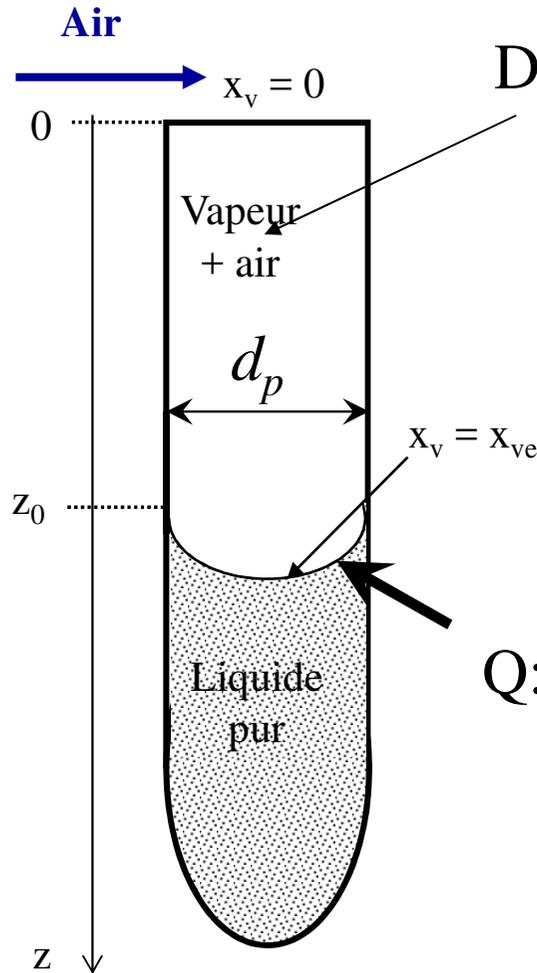

$$R_i = \alpha^{-1} \left(\frac{M_v}{2\pi RT} \right)^{-1/2}$$

$$\delta = \frac{D}{\alpha} \left[\frac{2\pi M_v}{RT} \right]^{1/2}$$

Longueur
de diffusion
équivalente

Q: Théorie établie pour une seule espèce. Impact du gaz inerte ?

Evaporation avec effet de raréfaction



Diffusion en espace confiné

$$D_{O_2}^a = \left(\frac{1}{D_{O_2-N_2}} + \frac{1}{D_{O_2,Kn}} \right)^{-1}$$

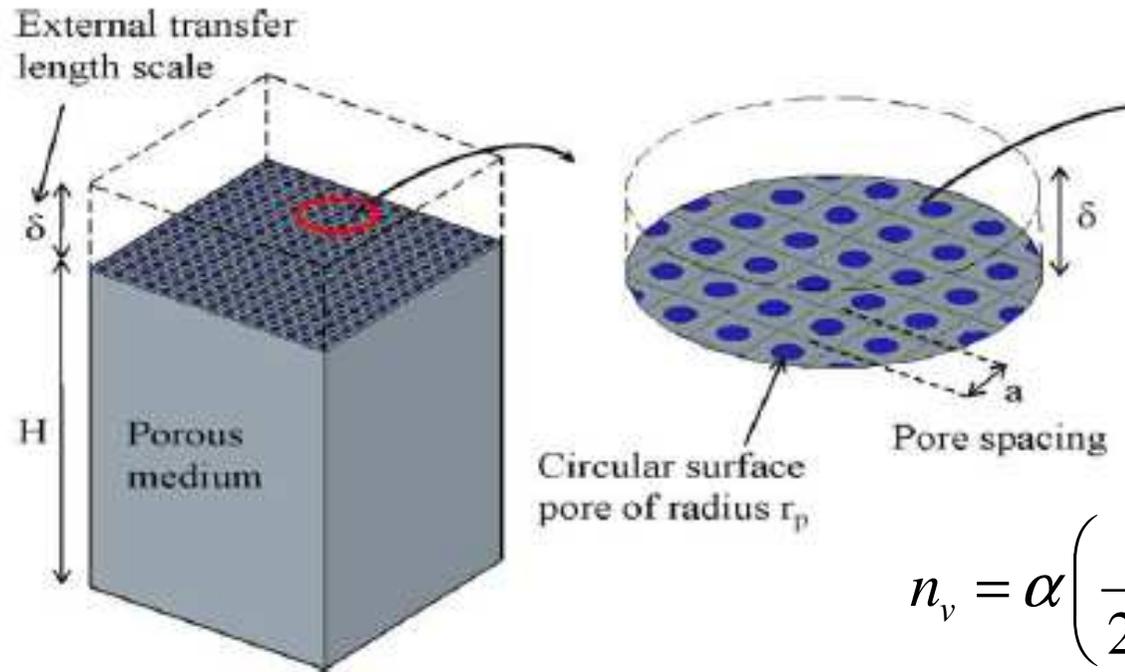
$$D_{O_2,Kn} = \frac{d_p}{3} \sqrt{\frac{8RT}{\pi M_{O_2}}}$$

Knudsen
Diffusivity
(dusty gas model)

Q: Théorie HKS: Impact du confinement ?

Q: modélisation de la diffusion ?

Théorie HKS et changement d'échelle ?



$$n_v = \alpha \left(\frac{M}{2\pi RT} \right)^{1/2} (P_{ve} - P_{vi})$$

Littérature:

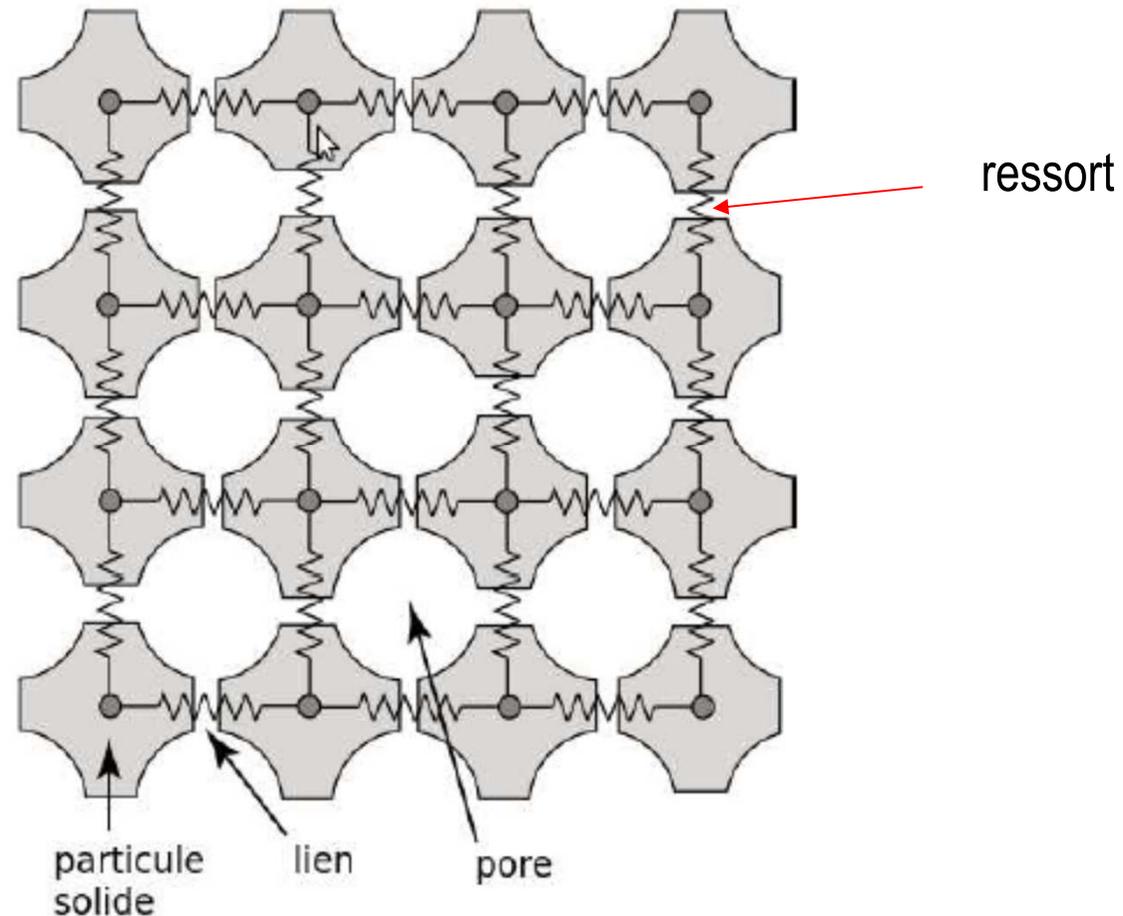
- nombreux exemples d'utilisation du modèle HKS directement à l'échelle macro.
- pas de justification théorique
- pas de validation expérimentale

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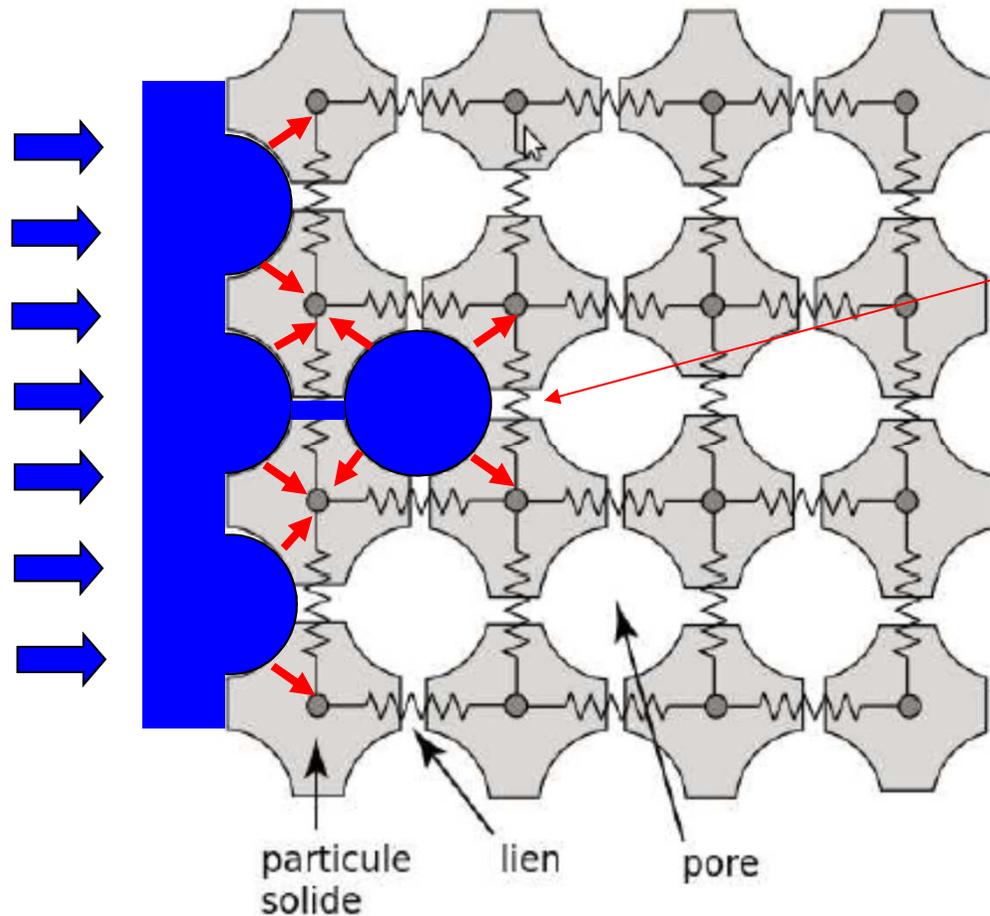
DRAINAGE HYPERLENT ET DEFORMATION MECANIQUE A PETITE ECHELLE

Modèle de réseau de pore classique couplé à un modèle de ressorts interconnectés



DRAINAGE HYPERLENT ET DEFORMATION MECANIQUE A PETITE ECHELLE

Origine de la déformation en drainage



Elargissement
des constrictions
induit par la
déformation

Capillarité \Rightarrow Pression fluide bleu \gg Pression fluide blanc

DRAINAGE HYPERLENT ET DEFORMATION MECANIQUE A PETITE ECHELLE

Paramètre clé: nombre de dilatance

$$N_f = \frac{\gamma}{\lambda a E^* \varepsilon_0^{1/2}}$$

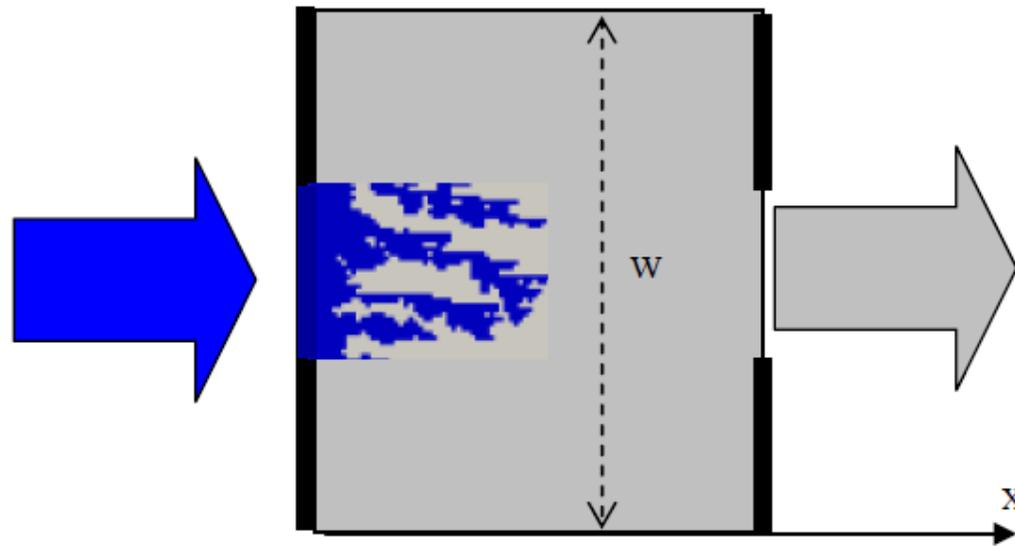
$N_f \ll 1$ déformation négligeable

$N_f \sim O(1)$ déformation (locale) marquée

N_f s'interprète comme le rapport entre l'incrément de la force pression nécessaire à l'envahissement d'un pore $\Delta f_p \sim \gamma a$ et l'incrément de force résultant de la déformation des particules $\Delta f_c \sim \lambda E^* a^2 \varepsilon_0^{1/2}$.

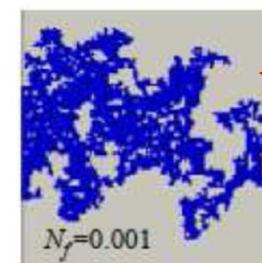
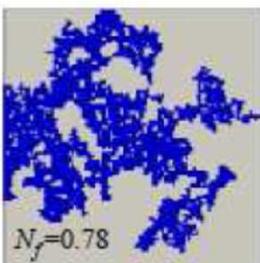
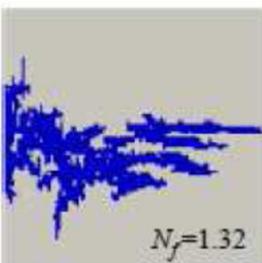
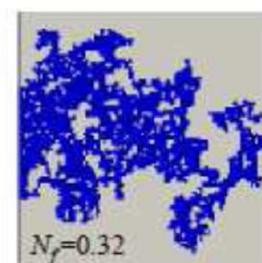
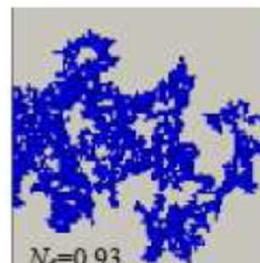
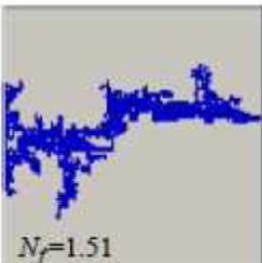
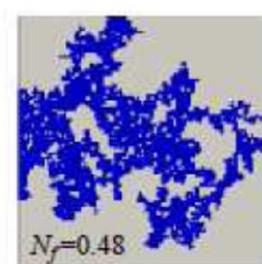
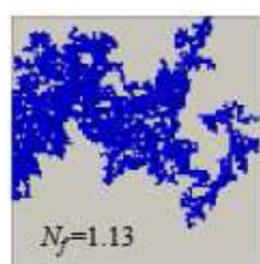
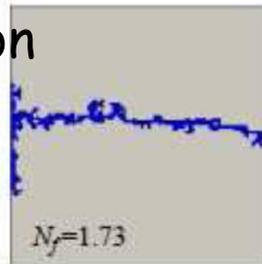
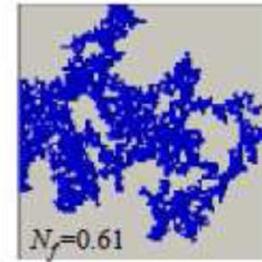
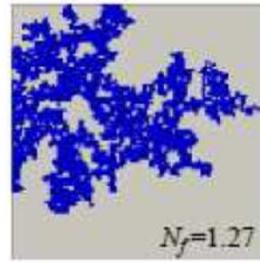
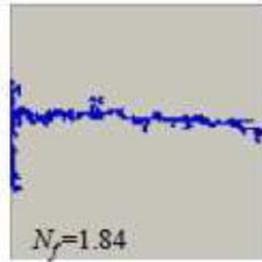
E^* = module d'Young effectif
 γ tension de surface

DRAINAGE HYPERLENT ET DEFORMATION MECANIQUE A PETITE ECHELLE



$$N_f \sim O(1)$$

Auto-génération
d'un chemin
préférentiel

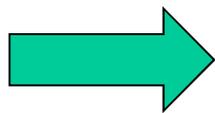
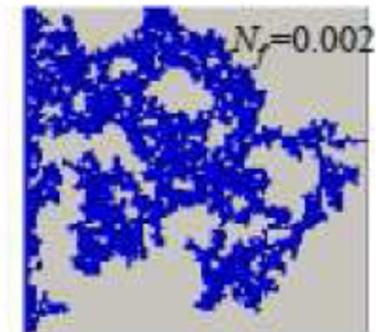
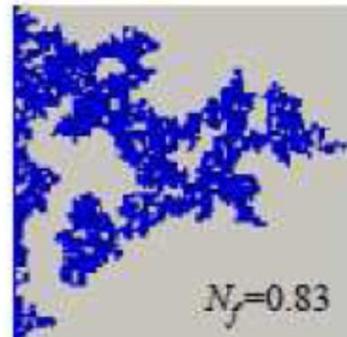
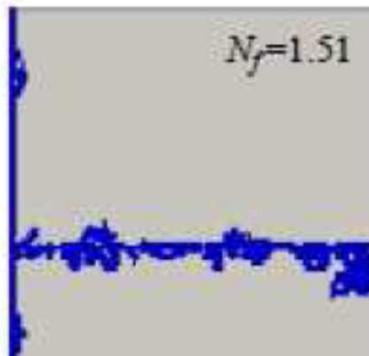


← Figure d'invasion
pour le milieu
rigide
 $N_f \ll 1$

DRAINAGE HYPERLENT ET DEFORMATION MECANIQUE A PETITE ECHELLE

Milieu « dilatant »

Le couplage fluide - structure à petite échelle peut changer profondément la figure d'invasion par rapport au cas de référence du milieu rigide



Couplage fluide –structure à l'échelle du V.E.R.

Merci pour votre attention